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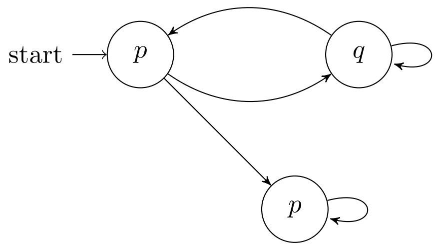
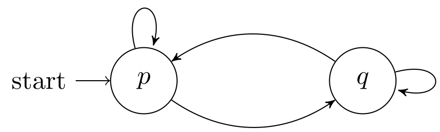
1. (2 points) Give LTL and CTL formulae that distinguish between the transition systems TS_1 and TS_2 over the set of atomic propositions $AP = \{p, q\}$. Clearly indicate which transition system satisfies which of the formulae. If such a formula does not exist (in LTL/CTL) then explain why.



TS_1



TS_2

2. Consider the formula $\varphi \equiv p \, U \bigcirc q$.
   (a) (1 point) Give the closure set $closure(\varphi)$.
   (b) (1 point) Enumerate the elementary sets of $closure(\varphi)$. Label the sets (Büchi states) as $q_1, q_2, \ldots$.
   (c) (1 point) Derive the generalized Büchi automaton that recognizes the models of φ using the elementary sets above. Give the transition relation between the states as an adjacency matrix with rows and columns labelled in the order $q_1, q_2, \ldots$ (left-to-right, top-to-bottom). Whenever there is an edge from a state q_i to a state q_j write the set of atomic propositions that marks the edge in the (i, j)'th cell else write $\emptyset$. Indicate initial and final states separately.

3. (5 points) Consider an LTL formula $\varphi \equiv (a \, U \, b) \rightarrow \Diamond b$. Is φ valid? If yes, give an automata-theoretic proof of validity (i.e., construct a suitable NBA and use nested DFS to check an appropriate persistence condition). Otherwise, give a transition system that violates the formula. Illustrate the constructions clearly.