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here: two propositional temporal logics:

LTL: linear temporal logic

CTL: computation tree logic

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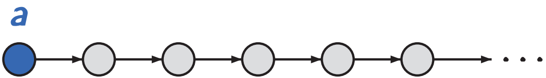

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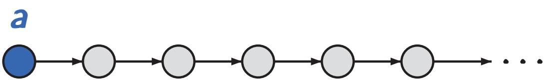

next operator
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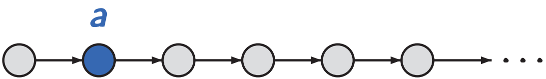

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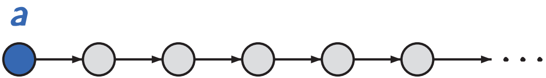

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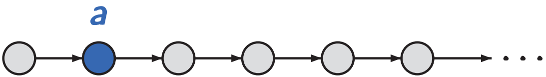

until operator
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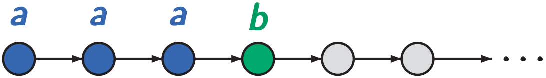

Derived operators in LTL

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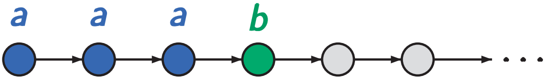

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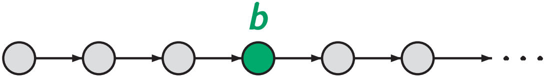

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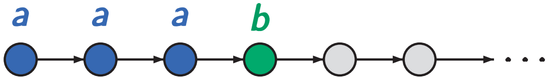

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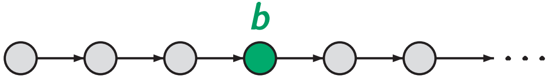
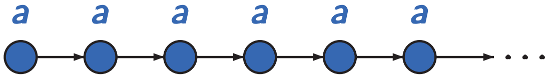


always $\Box a$



Next $\bigcirc$, until U and eventually $\Diamond$

$\Box\,(\text{try\_to\_send} \rightarrow \bigcirc\, \text{delivered})$

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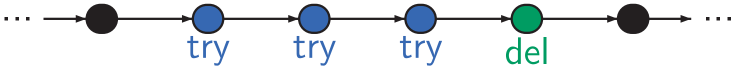

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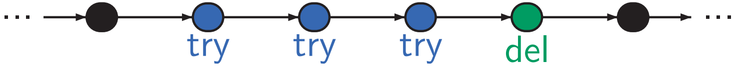

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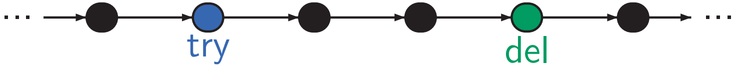

Examples for LTL formulas

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traffic light: $\Box(\mathit{yellow} \vee \bigcirc\neg \mathit{red})$

Infinitely often and eventually forever

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interpretation of **LTL** formulas over traces, i.e., infinite words over 2^{AP}

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LT property of formula φ:

$$Words(\varphi) \stackrel{\text{def}}{=} \{\sigma \in (2^{AP})^\omega : \sigma \models \varphi\}$$

LTL-semantics of derived operators $\Diamond$ and $\Box$

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$\sigma \models \Diamond\varphi$	iff	there exists $j \geq 0$ such that
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$\sigma \models \Box\varphi$	iff	for all $j \geq 0$ we have:
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LTL semantics over TS

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given a TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

define satisfaction relation $\models$ for

- **LTL** formulas over AP
- the maximal path fragments and states of $\mathcal{T}$

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assumption: $\mathcal{T}$ has no terminal states, i.e.,
all maximal path fragments in $\mathcal{T}$ are infinite

LTL semantics over paths of TS

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interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$

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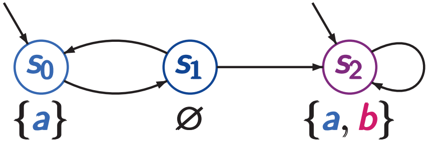
remind: LT property of an LTL formula:

$$Words(\varphi) = \{\sigma \in (2^{AP})^{\omega} : \sigma \models \varphi\}$$

Example: LTL-semantics over paths



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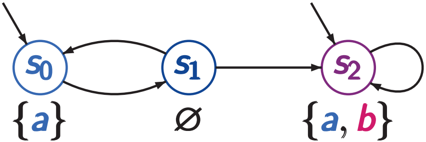

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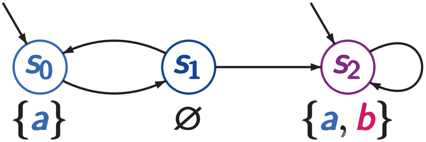

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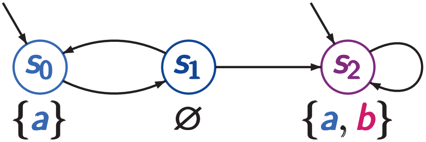

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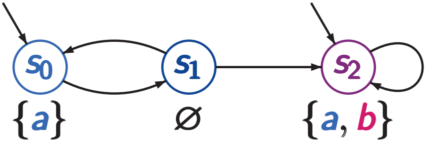
$\pi \models a$, but $\pi \not\models b$ $\quad$ as $L(s_0) = \{a\}$

Example: LTL-semantics over paths



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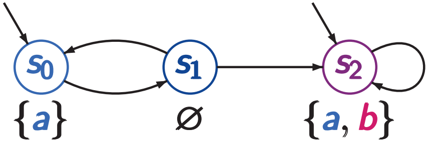

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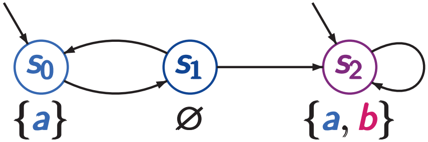

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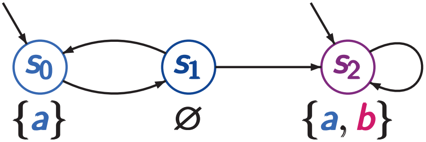

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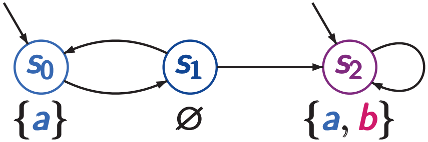

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$\pi \models \bigcirc(\neg a \wedge \neg b)$	as $L(s_1) = \varnothing$
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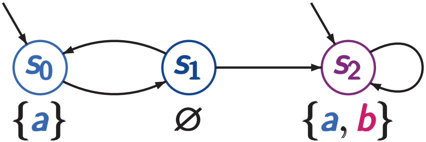

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$\pi \models (\neg b)\, \mathsf{U}\, (a \wedge b)$	as $s_0, s_1 \models \neg b$
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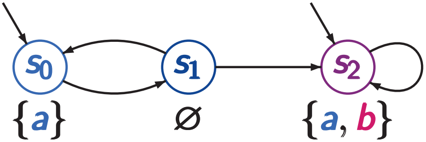

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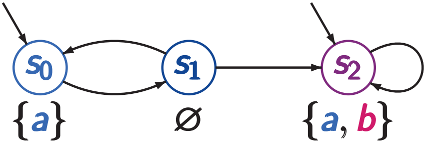
Correct or wrong ?



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path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1\, \ldots$

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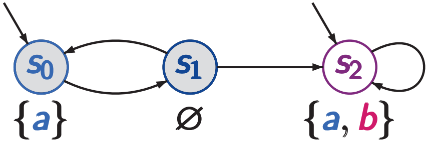

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path $\pi = s_0\, s_1\, s_0\, s_1\, s_0\, s_1 \ldots$

$trace(\pi) = (\{a\}\,\varnothing)^{\omega}$

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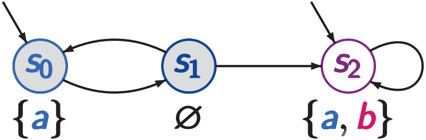

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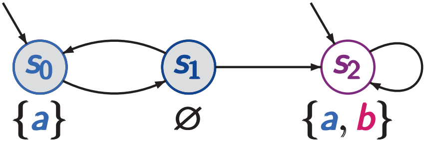

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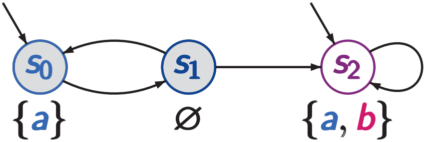

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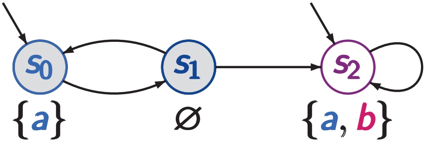
Correct or wrong ?



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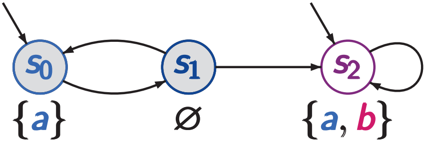

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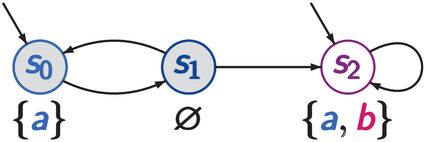

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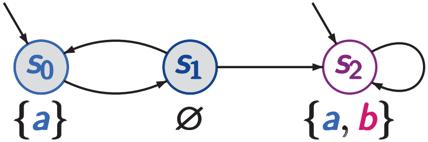

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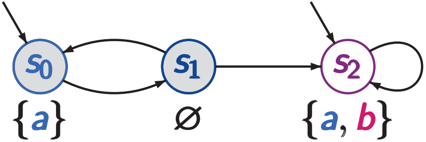
$\pi \models \Box a$?

Correct or wrong ?



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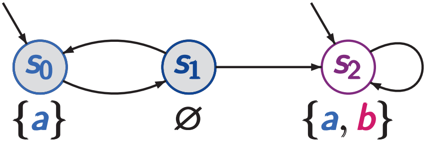
$\pi \not\models \Box a$ $\qquad$ as $s_1 \not\models a$



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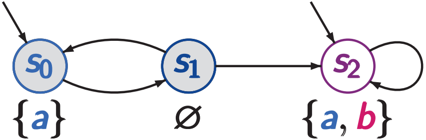
$\pi \models \Box\Diamond a$?

Correct or wrong ?



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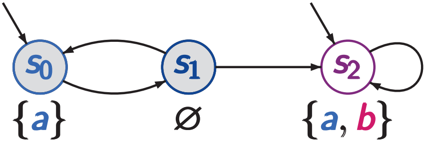

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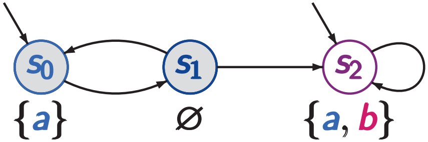

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$\pi \not\models a \,\mathsf{U}\, b$ as $s_0 \not\models b$ and $s_1 \not\models a \vee b$

$\pi \models \Diamond b \rightarrow (a \,\mathsf{U}\, b)$ as $\pi \not\models \Diamond b$

$\pi \models \bigcirc \bigcirc \neg b$ as $s_0 \models \neg b$

$\pi \not\models \Box a$ as $s_1 \not\models a$

$\pi \models \Box\Diamond a$ as $\Box\Diamond \mathrel{\widehat{=}}$ infinitely often

$\pi \not\models \Diamond\Box a$ as $\Diamond\Box \mathrel{\widehat{=}}$ eventually forever

LTL-semantics of derived operators

for $\sigma = A_0\, A_1\, A_2\, \ldots \in \left(2^{AP}\right)^{\omega}$:

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$\sigma \models \Box\varphi$	iff	for all $j \geq 0$ we have: $A_j A_{j+1} A_{j+2} \ldots \models \varphi$

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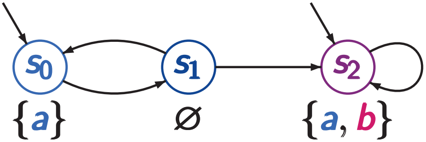
$\sigma \models \Box\Diamond\varphi$	iff	there are infinitely many $j \geq 0$ s.t. $A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$
$\sigma \models \Diamond\Box\varphi$	iff	for almost all $j \geq 0$ we have: $A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$

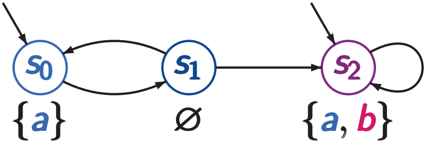
LTL-semantics over paths



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$

LTL-semantics over paths



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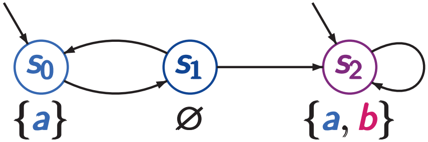

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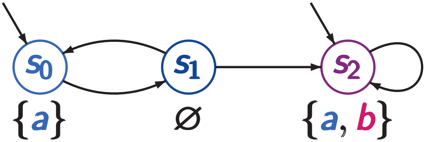
$\pi \models \bigcirc((\neg a \wedge \neg b)\, \mathsf{U}\, (a \wedge b))$?

LTL-semantics over paths



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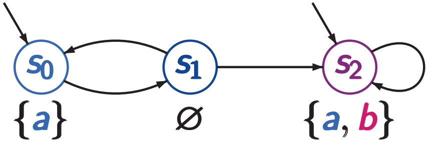
$\pi \models \bigcirc((\neg a \wedge \neg b)\, \mathsf{U}\, (a \wedge b))$ as $s_1 \models \neg a \wedge \neg b$

$s_2 \models a \wedge b$

LTL-semantics over paths



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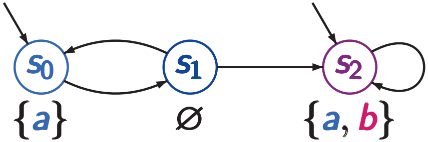
path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models \bigcirc((\neg a \wedge \neg b)\, \mathsf{U}\, (a \wedge b))$ as $s_1 \models \neg a \wedge \neg b$

$s_2 \models a \wedge b$

$\pi \models \bigcirc\Box(a \leftrightarrow b)$?



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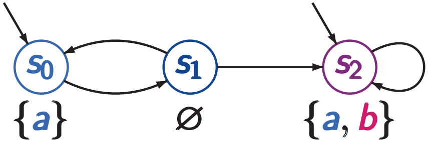
LTL-semantics over paths



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path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

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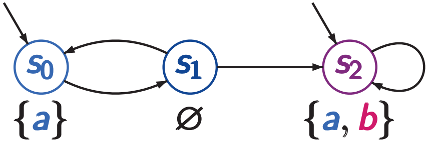
$s_2 \models a \wedge b$

$\pi \models \bigcirc\Box(a \leftrightarrow b)$ as $s_1, s_2 \models a \leftrightarrow b$

$\pi \models a\, \mathsf{U}\, (\neg b\, \mathsf{U}\, a)$?

LTL-semantics over paths



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path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \dots$ $\qquad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

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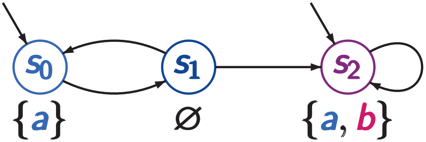
LTL-semantics over paths



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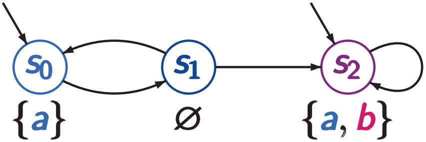

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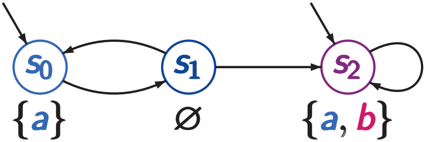

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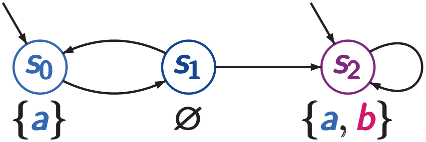

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LTL semantics over the states of a TS

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given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$

interpretation of φ over states:

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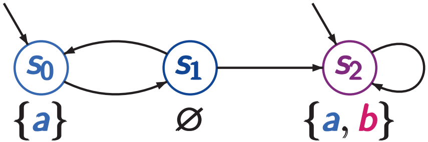
satisfaction relation for LT properties

Which formulas hold for $\mathcal{T}$?



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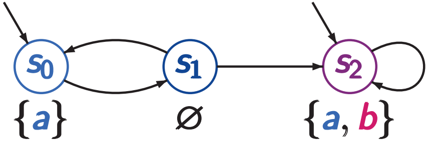

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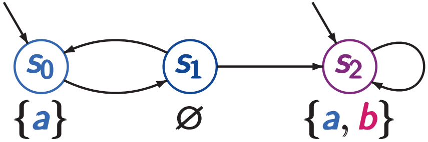

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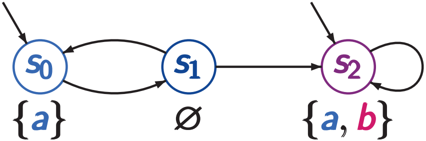

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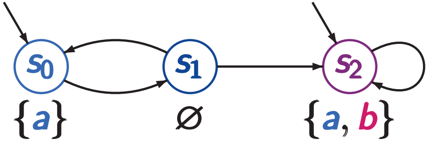

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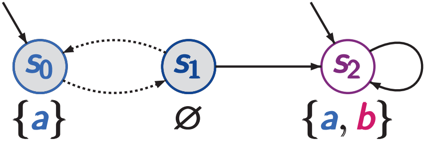

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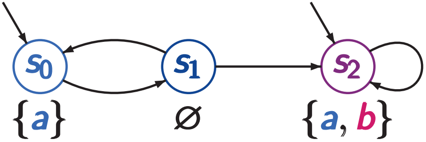

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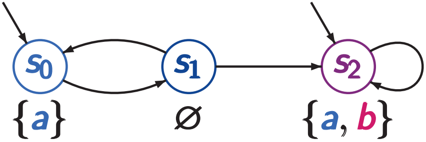

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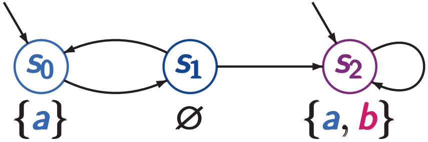

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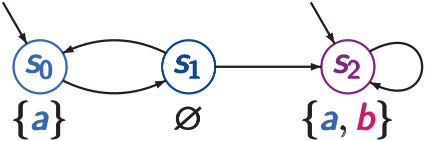

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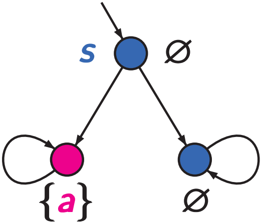

$s \not\models \Diamond a$ and $s \not\models \neg\Diamond a$

LTL-formulas for MUTEX protocols

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