LTL model checking

given: finite TS $\mathcal{T}$, LTL-formula φ
question: does $\mathcal{T} \models \varphi$ hold **?**

construct an NBA $\mathcal{A}$ for $\neg\varphi$ and the product $\mathcal{T} \otimes \mathcal{A}$

check whether $\mathcal{T} \otimes \mathcal{A} \models \Diamond\Box\neg F$ $\longleftarrow$ persistence checking nested **DFS**

IF $\mathcal{T} \otimes \mathcal{A} \models \Diamond\Box\neg F$

THEN return "**yes**"

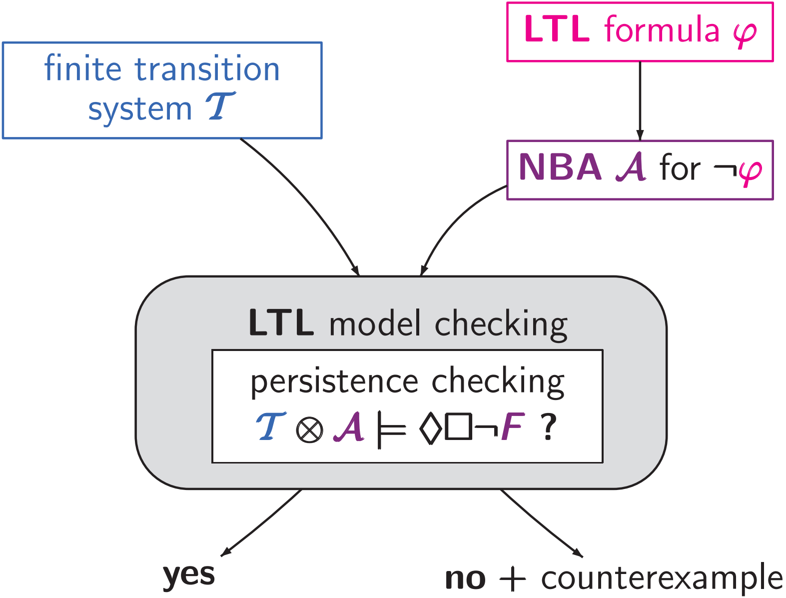
ELSE compute a counterexample

$\langle s_0, p_0\rangle \ldots \langle s_n, p_n\rangle \ldots \langle s_n, p_n\rangle$

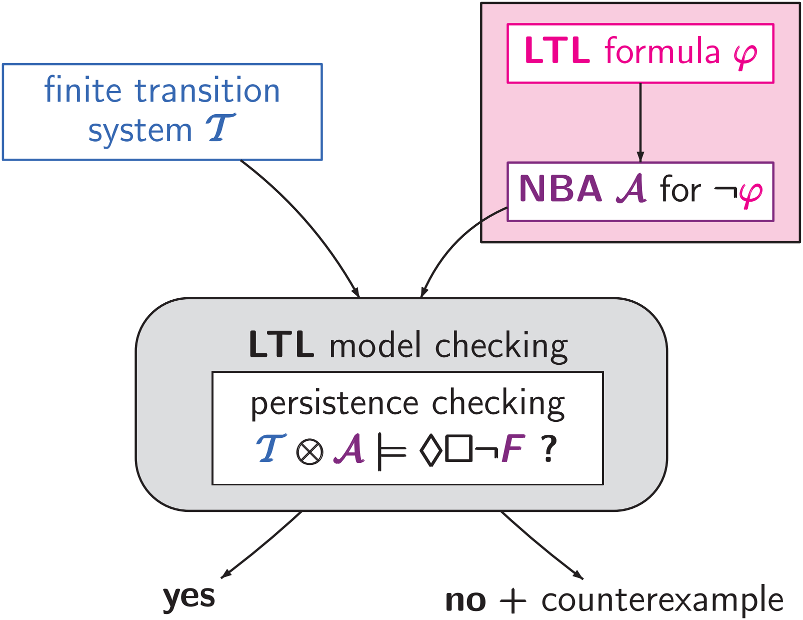
for $\mathcal{T} \otimes \mathcal{A}$ and $\Diamond\Box\neg F$

return "**no**" and $s_0 \ldots s_n \ldots s_n$

LTL model checking



LTL model checking



From LTL to NBA

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For each **LTL** formula φ there is an **NBA** $\mathcal{A}$ s.t.

$$\mathcal{L}_\omega(\mathcal{A}) \;=\; \mathit{Words}(\varphi)$$

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LTL formula φ

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NBA $\mathcal{A}$ s.t.
$\mathcal{L}_\omega(\mathcal{A}) = \mathit{Words}(\varphi)$

nondeterministic
Büchi automaton

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$$\mathcal{L}_\omega(\mathcal{A}) = \mathit{Words}(\varphi)$$

LTL formula φ

↓

GNBA $\mathcal{G}$ s.t. $\mathcal{L}_\omega(\mathcal{G}) = \mathit{Words}(\varphi)$ — generalized NBA, several acceptance sets

↓

NBA $\mathcal{A}$ s.t. $\mathcal{L}_\omega(\mathcal{A}) = \mathcal{L}_\omega(\mathcal{G})$ — nondeterministic Büchi automaton, **1** acceptance set

From LTL to NBA

For each **LTL** formula φ there is an **NBA** $\mathcal{A}$ s.t.

$$\mathcal{L}_\omega(\mathcal{A}) = \mathit{Words}(\varphi)$$

LTL formula φ

↓

GNBA $\mathcal{G}$ s.t.
$\mathcal{L}_\omega(\mathcal{G}) = \mathit{Words}(\varphi)$

↓

NBA $\mathcal{A}$ s.t.
$\mathcal{L}_\omega(\mathcal{A}) = \mathcal{L}_\omega(\mathcal{G})$

generalized NBA
k acceptance sets

↓ ***k*** copies of $\mathcal{G}$

nondeterministic
Büchi automaton
1 acceptance set

Encoding of LTL semantics in a GNBA

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idea: encode the semantics of the operators appearing in φ by appropriate components of the GNBA $\mathcal{G}$

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semantics of ...	encoding
propositional logic *true*, $\neg$, $\wedge$	
next $\bigcirc$	
until **U**	

Encoding of LTL semantics in a GNBA

idea: encode the semantics of the operators appearing in φ by appropriate components of the GNBA $\mathcal{G}$

semantics of ...	encoding
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next $\bigcirc$	
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idea: encode the semantics of the operators appearing in φ by appropriate components of the GNBA $\mathcal{G}$

semantics of ...	encoding
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until **U**	via expansion law

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$$\psi_1 \,\mathsf{U}\, \psi_2 \;\equiv\; \psi_2 \vee (\psi_1 \wedge \bigcirc(\psi_1 \,\mathsf{U}\, \psi_2))$$

idea: encode the semantics of the operators appearing in φ by appropriate components of the GNBA $\mathcal{G}$

semantics of ...	encoding
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$$\psi_1 \,\mathsf{U}\, \psi_2 \;\equiv\; \psi_2 \vee (\psi_1 \wedge \bigcirc(\psi_1 \,\mathsf{U}\, \psi_2))$$

encoded in the states (arrows point to ψ_2 and ψ_1)

Encoding of LTL semantics in a GNBA

idea: encode the semantics of the operators appearing in φ by appropriate components of the GNBA $\mathcal{G}$

semantics of ...	encoding
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next $\bigcirc$	in the transition relation
until **U**	via expansion law

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encoded in the states (ψ_2 and ψ_1)

encoded in the transition relation ($\bigcirc$)

Encoding of LTL semantics in a GNBA

idea: encode the semantics of the operators appearing in φ by appropriate components of the GNBA $\mathcal{G}$

semantics of ...	encoding
propositional logic ***true***, $\neg$, $\wedge$	in the states
next $\bigcirc$	in the transition relation
until **U**	expansion law, **least fixed point**

$$\psi_1 \, \mathsf{U} \, \psi_2 \;\equiv\; \psi_2 \vee (\psi_1 \wedge \bigcirc(\psi_1 \, \mathsf{U} \, \psi_2))$$

- ψ_2, ψ_1: encoded in the states
- $\bigcirc$: encoded in the transition relation
- least fixed point: acceptance condition

LTL $\rightsquigarrow$ GNBA

LTL formula $\varphi \rightsquigarrow$ GNBA $\mathcal{G}$ for ***Words***(φ)

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states of $\mathcal{G}$ $\widehat{=}$ (certain) sets of subformulas of φ s.t. each word $\sigma = A_0 A_1 A_2 \ldots \in Words(\varphi)$ can be extended to an accepting run $B_0 B_1 B_2 \ldots$ in $\mathcal{G}$

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$$A_0 \quad A_1 \quad A_2 \quad A_3 \quad \ldots \quad \in \mathit{Words}(\varphi)$$

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$$\begin{array}{cccccl} A_0 & A_1 & A_2 & A_3 & \ldots & \in Words(\varphi) \\ \downarrow & \downarrow & \downarrow & \downarrow & & \\ B_0 & B_1 & B_2 & B_3 & \ldots & \text{accepting run} \end{array}$$

where $B_i = \{\psi \in cl(\varphi) : A_i A_{i+1} A_{i+2} \ldots \models \psi\}$

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$\uparrow$ set of subformulas of φ and their negations

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Example: $\varphi = a \, \mathsf{U} (\neg a \wedge b)$

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$$
\begin{array}{ccccccc}
\{a\} & \{a\} & \{a,b\} & \{b\} & \varnothing & \varnothing & \ldots \models \varphi \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\
B_0 & B_1 & B_2 & B_3 & B_4 & B_5 &
\end{array}
$$

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\begin{array}{ccccccc}
\{a\} & \{a\} & \{a,b\} & \{b\} & \varnothing & \varnothing & \ldots \models \varphi \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\
B_0 & B_1 & B_2 & B_3 & B_4 & B_5 &
\end{array}
$$

where the B_i's are subsets of $\{a, \neg a, b, \neg b, \psi, \neg\psi, \varphi, \neg\varphi\}$

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$$\{a\} \quad \{a\} \quad \{a, b\} \quad \{b\} \quad \varnothing \quad \varnothing \quad \ldots \models \varphi$$

$$\downarrow$$

$$\begin{pmatrix} a \\ \neg b \\ \neg \psi \\ \varphi \end{pmatrix}$$

just for better readability:
tuple rather than set notation

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$\downarrow \qquad \downarrow$

$\{a\}$	$\{a\}$
a, $\neg b$, $\neg\psi$, φ	a, $\neg b$, $\neg\psi$, φ

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$\{a\}$	$\{a\}$	$\{a, b\}$
$\downarrow$	$\downarrow$	$\downarrow$
a	a	a
$\neg b$	$\neg b$	b
$\neg \psi$	$\neg \psi$	$\neg \psi$
φ	φ	φ

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$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$				
a $\neg b$ $\neg\psi$ φ	a $\neg b$ $\neg\psi$ φ	a b $\neg\psi$ φ	$\neg a$ b ψ φ				

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$\{a\}$	$\{a\}$	$\{a, b\}$	$\{b\}$	$\varnothing$	$\varnothing$	$\ldots$	$\models \varphi$
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$			
a	a	a	$\neg a$	$\neg a$			
$\neg b$	$\neg b$	b	b	$\neg b$			
$\neg\psi$	$\neg\psi$	$\neg\psi$	ψ	$\neg\psi$			
φ	φ	φ	φ	$\neg\varphi$			

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$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$		
a, $\neg b$, $\neg\psi$, φ	a, $\neg b$, $\neg\psi$, φ	a, b, $\neg\psi$, φ	$\neg a$, b, ψ, φ	$\neg a$, $\neg b$, $\neg\psi$, $\neg\varphi$	$\neg a$, $\neg b$, $\neg\psi$, $\neg\varphi$	$\ldots$	

Closure of LTL formulas

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Let φ be an LTL formula. Then:

$$\mathit{subf}(\varphi) \stackrel{\text{def}}{=} \text{set of all subformulas of } \varphi$$

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where ψ and $\neg\neg\psi$ are identified

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Example: if $\varphi = a \, \mathsf{U}(\neg a \wedge b)$ then

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Example: if $\varphi' = \Box a$

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Example: if $\varphi' = \Box a = \neg\Diamond\neg a = \neg(true\,\mathsf{U}\,\neg a)$ then

$$cl(\varphi') = \{a, \neg a, true, \neg true, \Box a, \neg\Box a\}$$

Elementary formula-sets

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(1) B is consistent w.r.t. propositional logic

(2) B is maximal consistent

(3) B is locally consistent with respect to until $\mathbf{U}$:

Elementary formula-sets

Let $B \subseteq cl(\varphi)$. B is called elementary if:

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if $\psi \in B$ then $\neg\psi \notin B$

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if $\psi_1 \wedge \psi_2 \in B$ then $\neg\psi_1 \notin B$ and $\neg\psi_2 \notin B$

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if $\psi_1 \in B$ and $\psi_2 \in B$ then $\neg(\psi_1 \wedge \psi_2) \notin B$

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if $\psi_1 \in B$ and $\psi_2 \in B$ then $\neg(\psi_1 \wedge \psi_2) \notin B$
if $\mathit{false} \in cl(\varphi)$ then $\mathit{false} \notin B$

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if $\mathit{false} \in cl(\varphi)$ then $\mathit{false} \notin B$

(2) B is maximal consistent

if $\psi \in cl(\varphi) \setminus B$ then $\neg\psi \in B$

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Let $B \subseteq cl(\varphi)$. B is called elementary if:

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(3) B is locally consistent with respect to until $\mathbf{U}$:

if $\psi_1 \,\mathbf{U}\, \psi_2 \in B$ and $\neg\psi_2 \in B$ then $\neg\psi_1 \notin B$

Elementary formula-sets

Let $B \subseteq cl(\varphi)$. B is called elementary if:

(1) B is consistent w.r.t. propositional logic

if $\psi \in B$ then $\neg\psi \notin B$
if $\psi_1 \wedge \psi_2 \in B$ then $\neg\psi_1 \notin B$ and $\neg\psi_2 \notin B$
if $\psi_1 \in B$ and $\psi_2 \in B$ then $\neg(\psi_1 \wedge \psi_2) \notin B$
if $\mathit{false} \in cl(\varphi)$ then $\mathit{false} \notin B$

(2) B is maximal consistent

if $\psi \in cl(\varphi) \setminus B$ then $\neg\psi \in B$

(3) B is locally consistent with respect to until $\mathbf{U}$:

if $\psi_1 \, \mathbf{U} \, \psi_2 \in B$ and $\neg\psi_2 \in B$ then $\neg\psi_1 \notin B$

if $\psi_2 \in B$ and $\psi_1 \, \mathbf{U} \, \psi_2 \in cl(\varphi)$ then $\neg(\psi_1 \, \mathbf{U} \, \psi_2) \notin B$

Elementary formula-sets

$B \subseteq cl(\varphi)$ is elementary iff:

(i) B is maximal consistent w.r.t. prop. logic,
i.e., if ψ, $\psi_1 \wedge \psi_2 \in cl(\varphi)$ then:

$$\begin{array}{rcl} \psi \notin B & \text{iff} & \neg\psi \in B \\ \psi_1 \wedge \psi_2 \in B & \text{iff} & \psi_1 \in B \text{ and } \psi_2 \in B \\ \mathit{true} \in cl(\varphi) & \text{implies} & \mathit{true} \in B \end{array}$$

(ii) B is locally consistent with respect to until **U**,
i.e., if $\psi_1 \mathbf{U} \psi_2 \in cl(\varphi)$ then:

$$\begin{array}{l} \text{if } \psi_1 \mathbf{U} \psi_2 \in B \text{ and } \psi_2 \notin B \text{ then } \psi_1 \in B \\ \text{if } \psi_2 \in B \text{ then } \psi_1 \mathbf{U} \psi_2 \in B \end{array}$$

Elementary or not?

Let $\varphi = a \, \mathsf{U}(\neg a \wedge b)$.

$B_1 = \{a, b, \neg a \wedge b, \varphi\}$

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$B_1 = \{a, b, \neg a \wedge b, \varphi\}$ not elementary

propositional inconsistent

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$B_2 = \{\neg a, b, \varphi\}$

Elementary or not?

Let $\varphi = a \, \mathsf{U} (\neg a \wedge b)$.

$B_1 = \{a, b, \neg a \wedge b, \varphi\}$ not elementary
propositional inconsistent

$B_2 = \{\neg a, b, \varphi\}$ not elementary, not maximal
as $\neg a \wedge b \notin B_2$
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Elementary or not?

Let $\varphi = a \, \mathsf{U} (\neg a \wedge b)$.

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$B_4 = \{\neg a, \neg b, \neg(\neg a \wedge b), \neg\varphi\}$ elementary

Example: elementary formula-sets

closure $cl(\varphi)$:

- set of all subformulas of φ and their negations
- ψ and $\neg\neg\psi$ are identified

elementary formula-sets: subsets B of $cl(\varphi)$

- maximal consistent w.r.t. propositional logic
- locally consistent w.r.t. U

For $\varphi = a\,\mathsf{U}(\neg a \wedge b)$, the elementary sets are:

$\{\ a,\ \ b, \neg(\neg a \wedge b), \varphi\}$ $\qquad$ $\{\ a,\ \ b, \neg(\neg a \wedge b), \neg\varphi\}$

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Encoding of LTL semantics in a GNBA

idea: encode the semantics of the operators appearing in φ by appropriate components of the GNBA $\mathcal{G}$:

semantics of ...	encoding
propositional logic *true*, $\neg$, $\wedge$	in the states
next $\bigcirc$	in the transition relation
until **U**	expansion law, least fixed point

$$\psi_1 \,\mathsf{U}\, \psi_2 \;\equiv\; \psi_2 \vee (\psi_1 \wedge \bigcirc(\psi_1 \,\mathsf{U}\, \psi_2))$$

encoded in the states (ψ_2, ψ_1) | encoded in the transition relation ($\bigcirc$) | acceptance condition (least fixed point)

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- ψ_2, ψ_1: elementary formula sets
- $\bigcirc$: encoded in the transition relation
- least fixed point: acceptance condition

GNBA for LTL-formula φ

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Example: GNBA for $\varphi = \bigcirc a$

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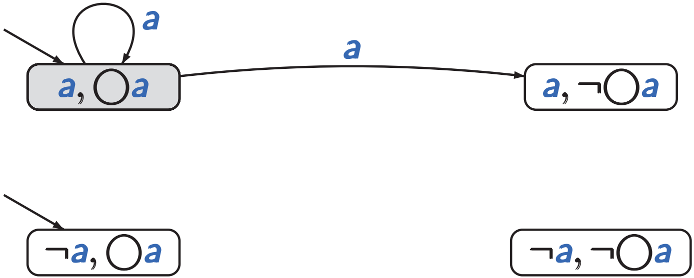

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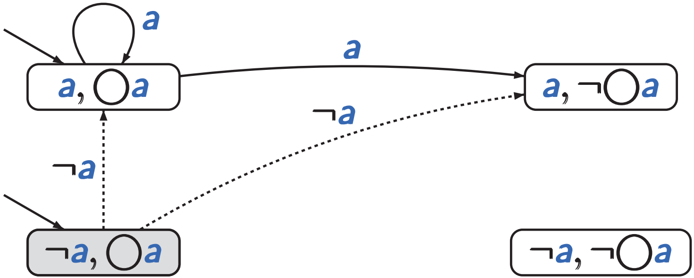

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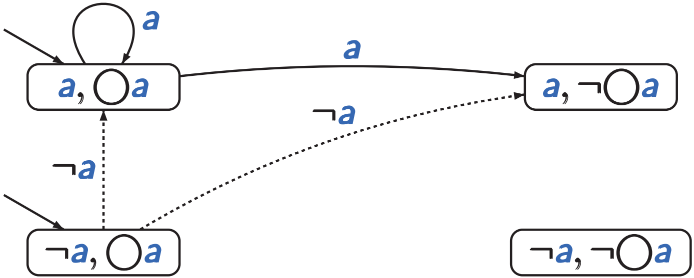

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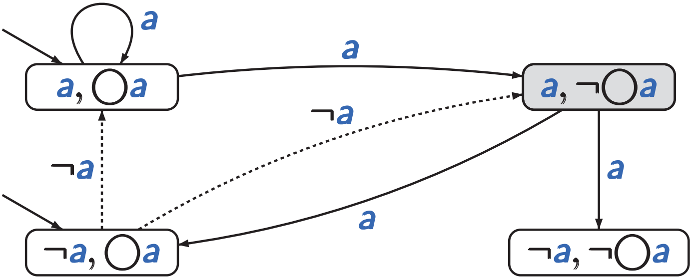

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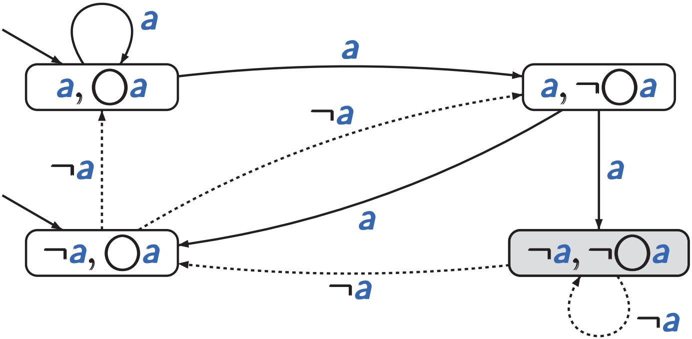

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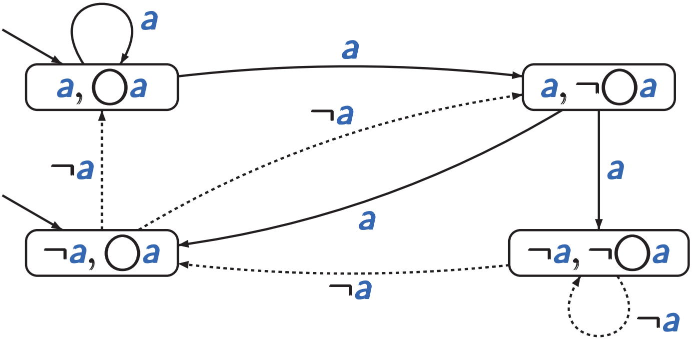
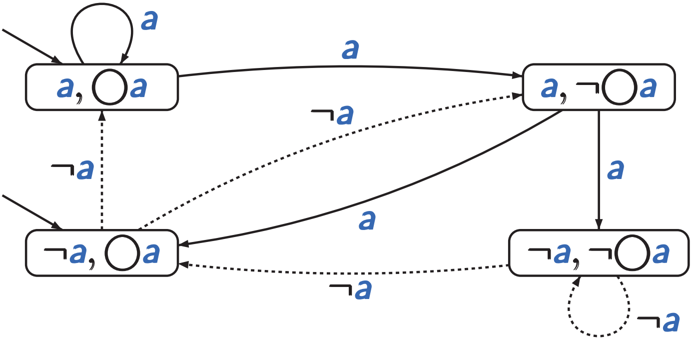


set of acceptance sets:

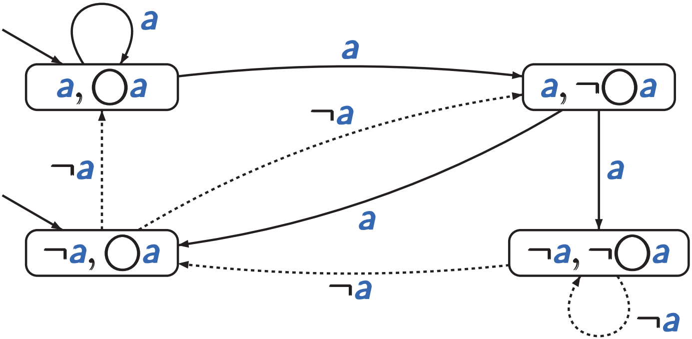
Example: GNBA for $\varphi = \bigcirc a$



set of acceptance sets: $\mathcal{F} = \varnothing$

hence: all words having an infinite run are accepted

Example: GNBA for $\varphi = \bigcirc a$



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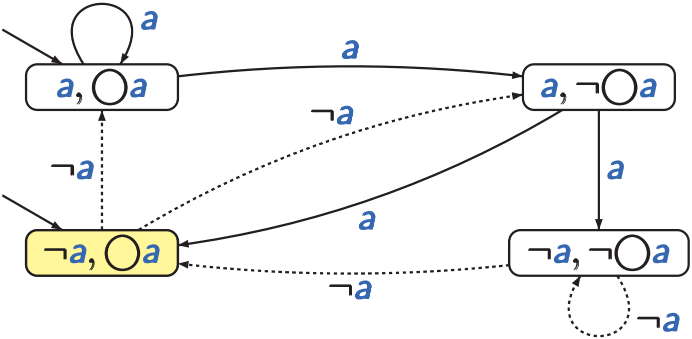

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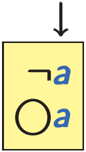

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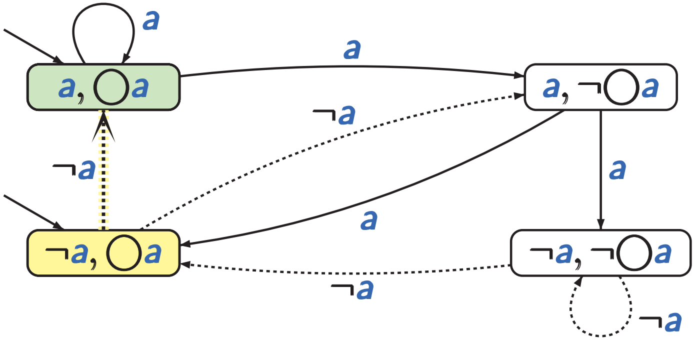

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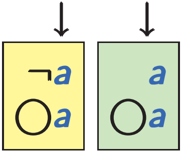

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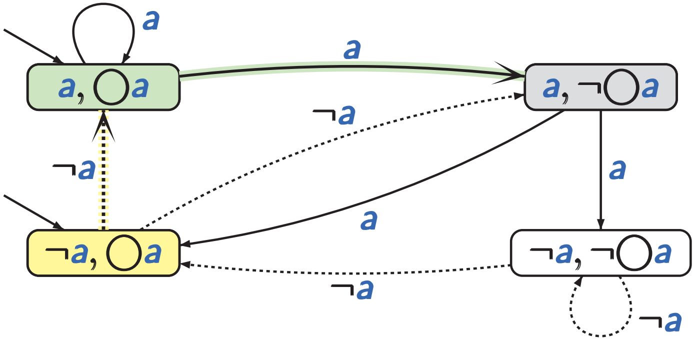

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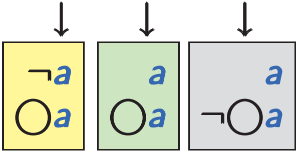

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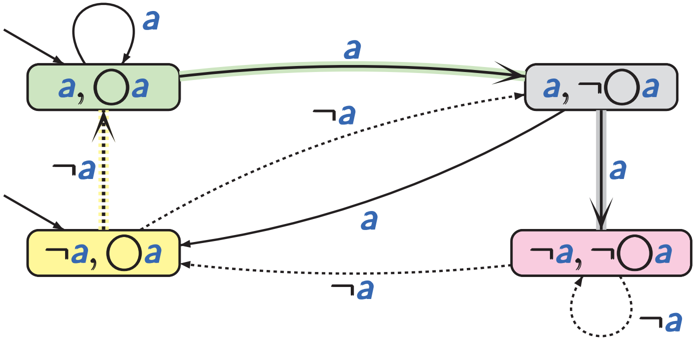

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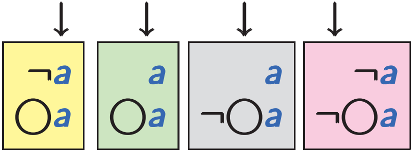

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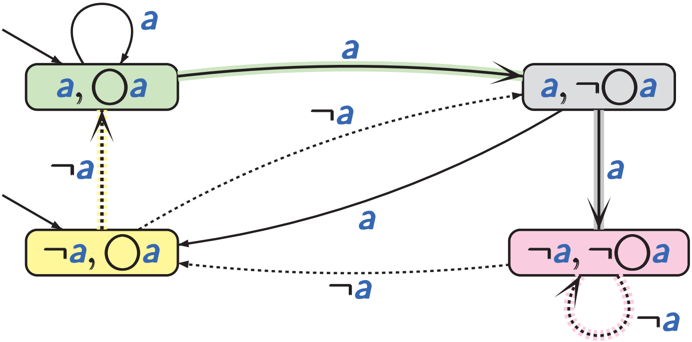

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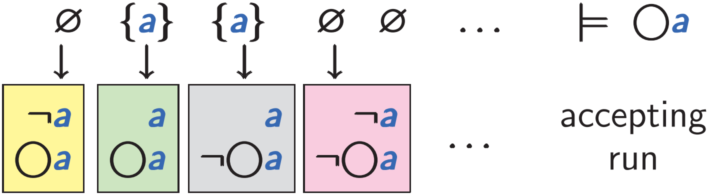
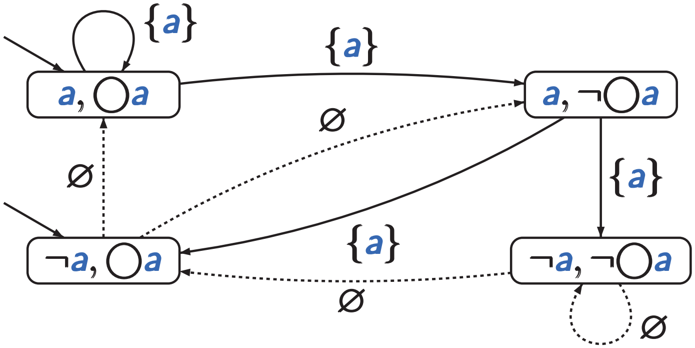
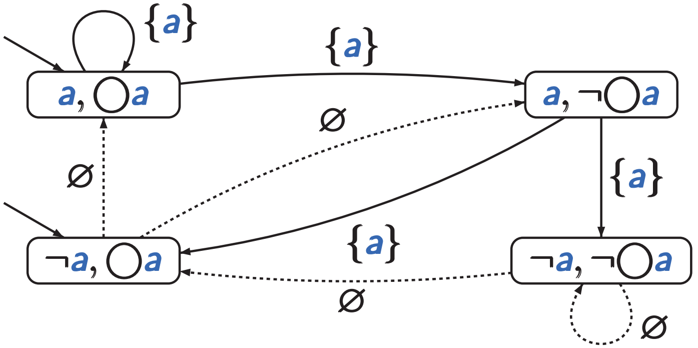


Soundness of the GNBA for $\varphi = \bigcirc a$



for all words $\sigma = A_0\, A_1\, A_2\, A_3 \ldots \in \mathcal{L}_\omega(\mathcal{G})$: $\quad A_1 = \{a\}$

Soundness of the GNBA for $\varphi = \bigcirc a$



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proof:

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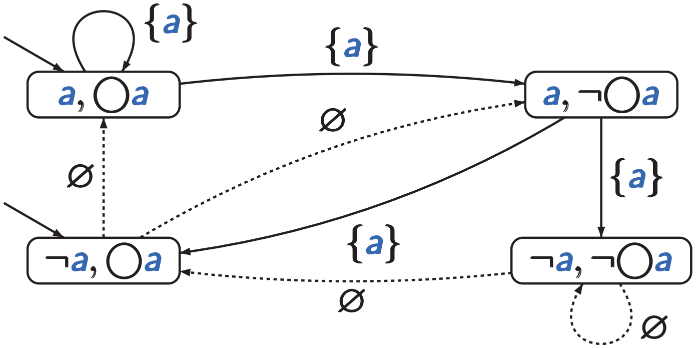

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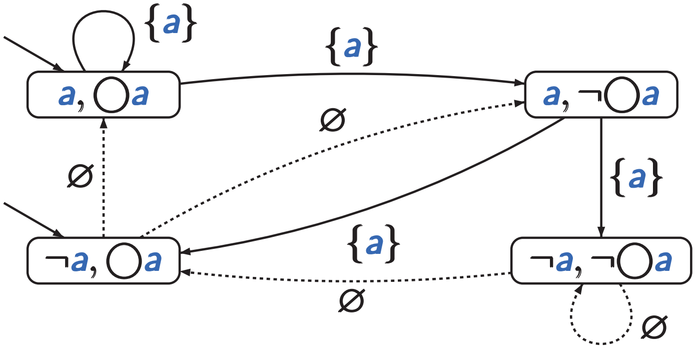

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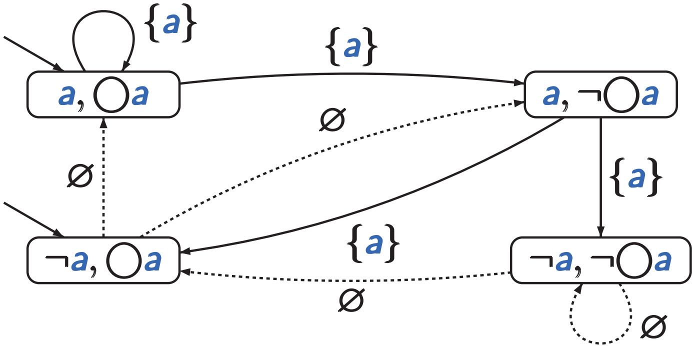

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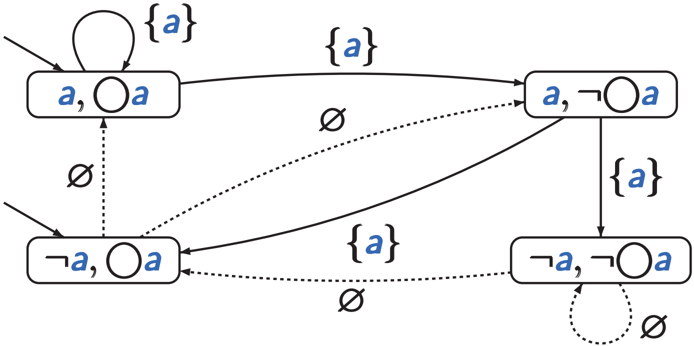

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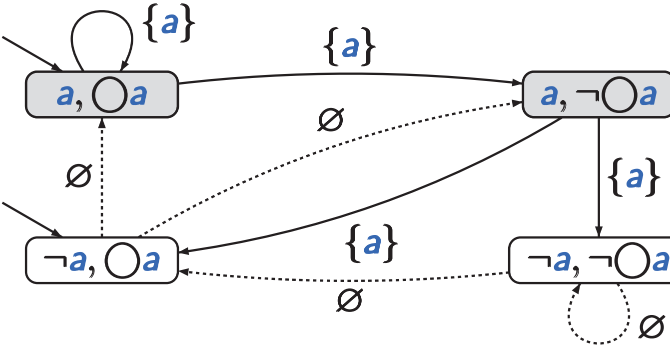

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$\Longrightarrow$ $\{a\} = B_1 \cap AP = A_1$

Example: GNBA for $\varphi = a \, \mathsf{U} \, b$

Example: GNBA for $\varphi = a \mathsf{U} b$

$a, b, a \mathsf{U} b$

$\neg a, \neg b, \neg(a \mathsf{U} b)$

$a, \neg b, a \mathsf{U} b$

$a, \neg b, \neg(a \mathsf{U} b)$

$\neg a, b, a \mathsf{U} b$

locally inconsistent: $\{a, b, \neg(a \mathsf{U} b)\}$

$\{\neg a, b, \neg(a \mathsf{U} b)\}$

$\{\neg a, \neg b, a \mathsf{U} b\}$

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$a, \neg b, a \, \mathsf{U} \, b$

$a, \neg b, \neg(a \, \mathsf{U} \, b)$

$\neg a, b, a \, \mathsf{U} \, b$

initial states: B with $\varphi = a \, \mathsf{U} \, b \in B$

Example: GNBA for $\varphi = a \, \mathsf{U} \, b$

$\longrightarrow$ $a, b, a \, \mathsf{U} \, b$

$\neg a, \neg b, \neg(a \, \mathsf{U} \, b)$

$\longrightarrow$ $a, \neg b, a \, \mathsf{U} \, b$

$a, \neg b, \neg(a \, \mathsf{U} \, b)$

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$\longrightarrow$ $\neg a, b, a \,\mathsf{U}\, b$

initial states: B with $\varphi = a \,\mathsf{U}\, b \in B$

acceptance condition: just one set of accept states

F = set of all B with $\varphi \notin B$ or $b \in B$

Example: GNBA for $\varphi = a \, \mathsf{U} \, b$ ⟵ NBA

$\longrightarrow$ $a, b, a \, \mathsf{U} \, b$

$\neg a, \neg b, \neg(a \, \mathsf{U} \, b)$

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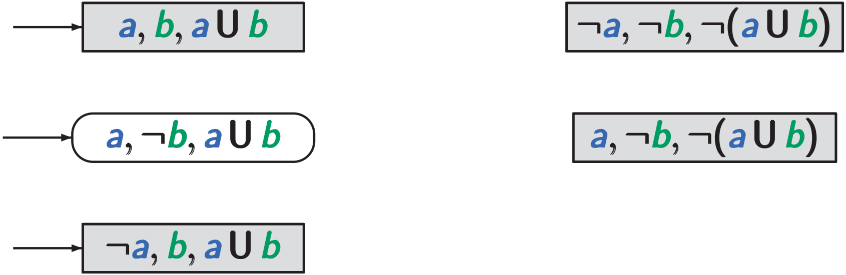
Example: (G)NBA for $\varphi = a \,\mathsf{U}\, b$



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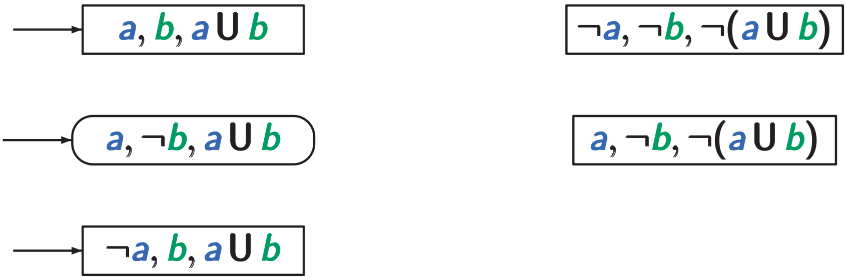
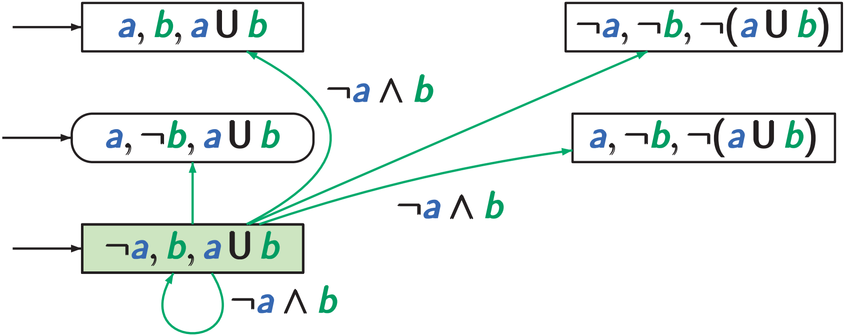


transition relation: $B' \in \delta(B, B \cap AP)$ iff

$$a \,\mathsf{U}\, b \in B \iff \big(b \in B \;\vee\; (\,a \in B \;\wedge\; a \,\mathsf{U}\, b \in B'\,)\big)$$

Example: (G)NBA for $\varphi = a \mathsf{U} b$



transition relation: $B' \in \delta(B, B \cap AP)$ iff

$$a \mathsf{U} b \in B \iff \big(b \in B \vee (a \in B \wedge a \mathsf{U} b \in B')\big)$$

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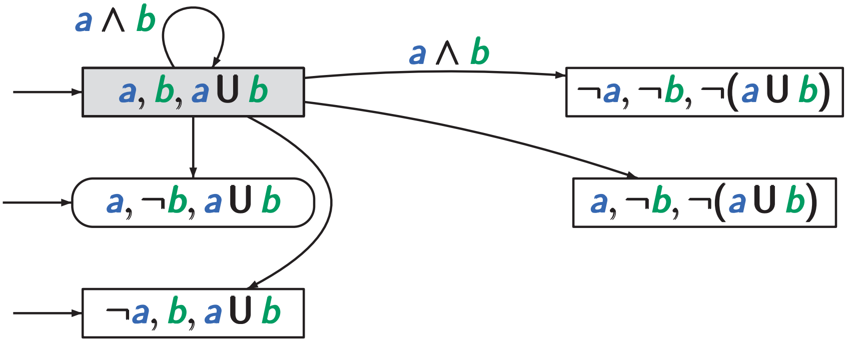

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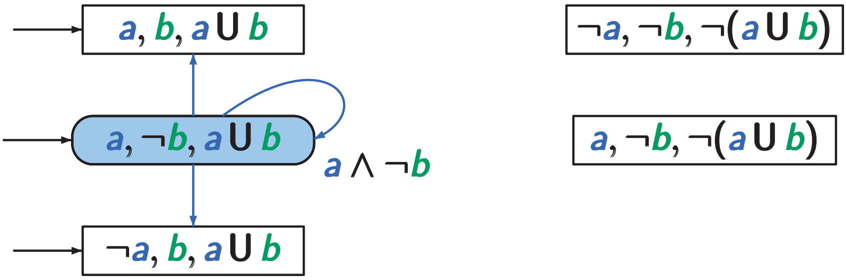

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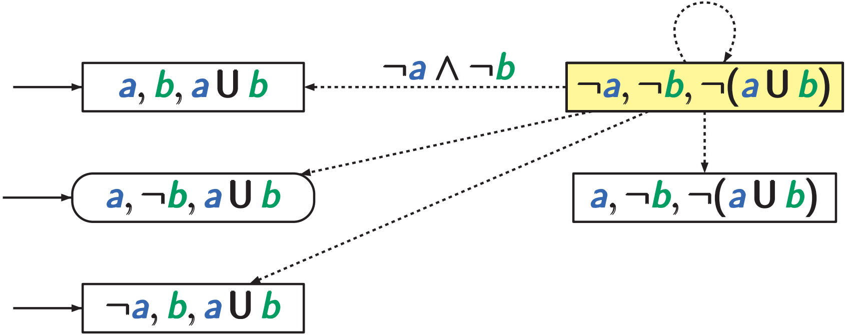

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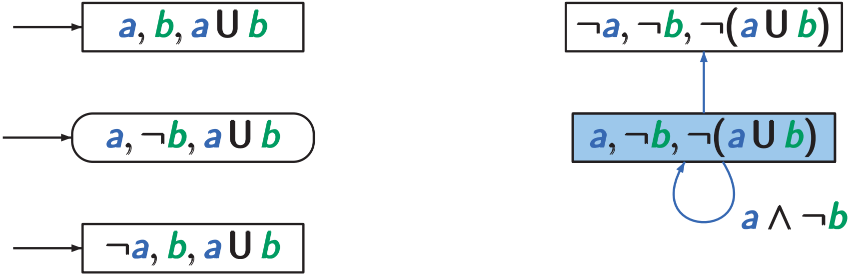

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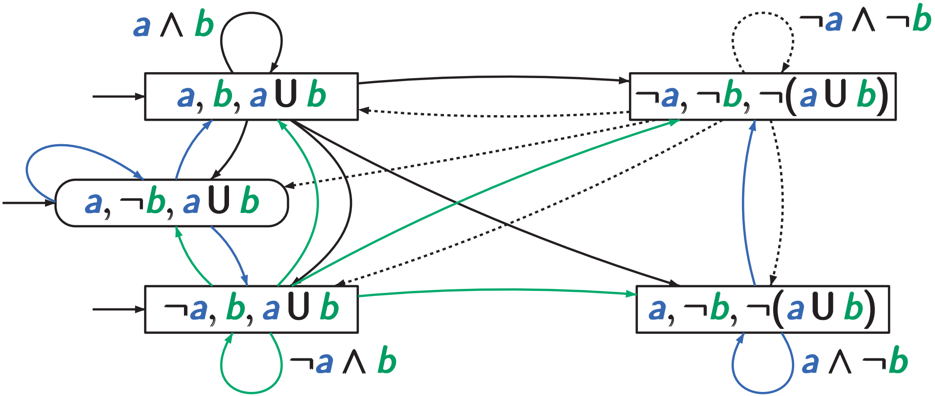
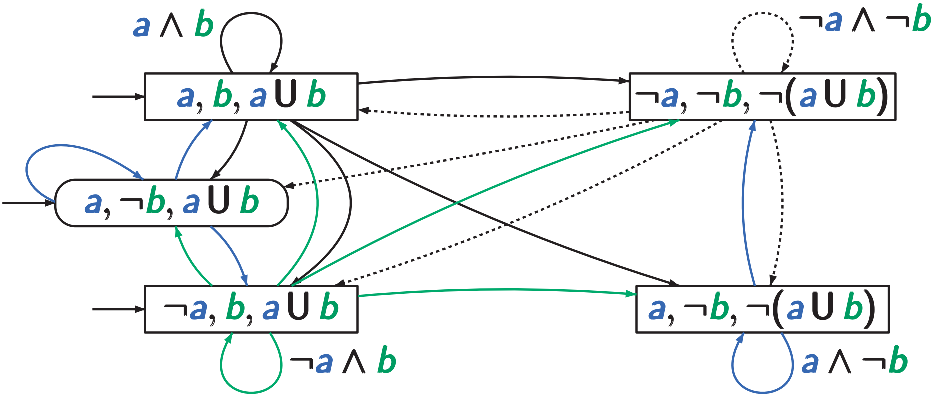
Example: (G)NBA for $\varphi = a \cup b$



Example: (G)NBA for $\varphi = a \,\mathsf{U}\, b$



$\{a\} \quad \{a\} \quad \{a, b\} \quad \varnothing \quad \varnothing \quad \varnothing \quad \ldots \models a \,\mathsf{U}\, b$

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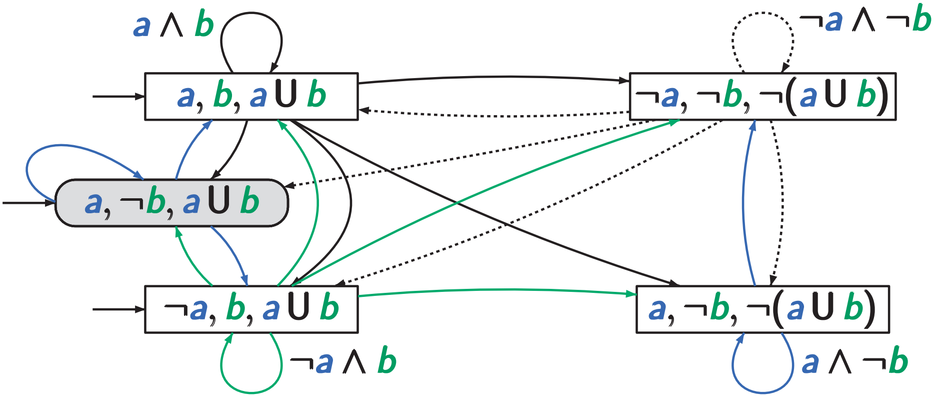
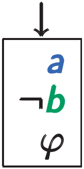
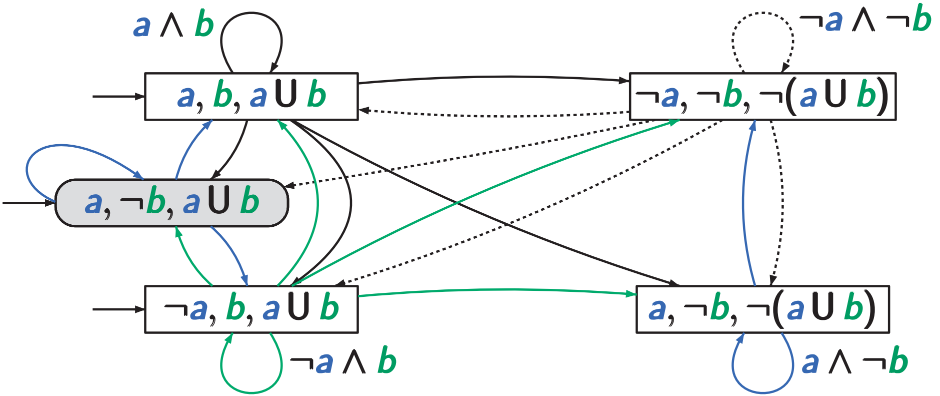


$\{a\} \quad \{a\} \quad \{a, b\} \quad \varnothing \quad \varnothing \quad \varnothing \quad \ldots \models a \,\mathsf{U}\, b$



Example: (G)NBA for $\varphi = a \mathsf{U} b$



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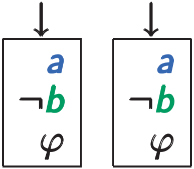
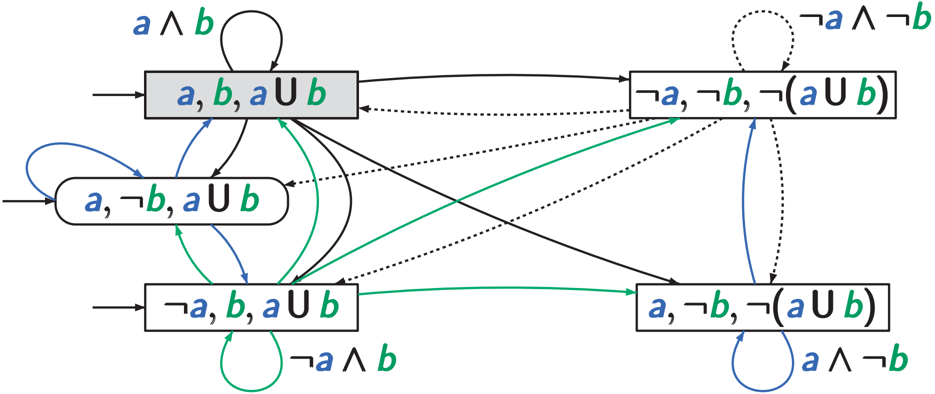


Example: (G)NBA for $\varphi = a \,\mathsf{U}\, b$



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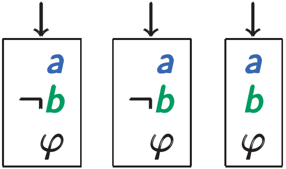
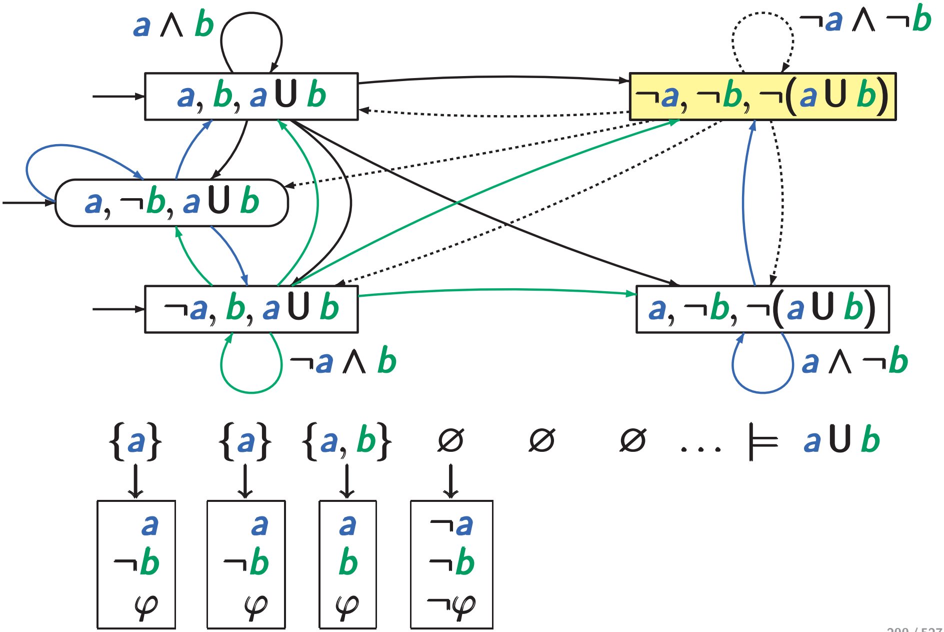
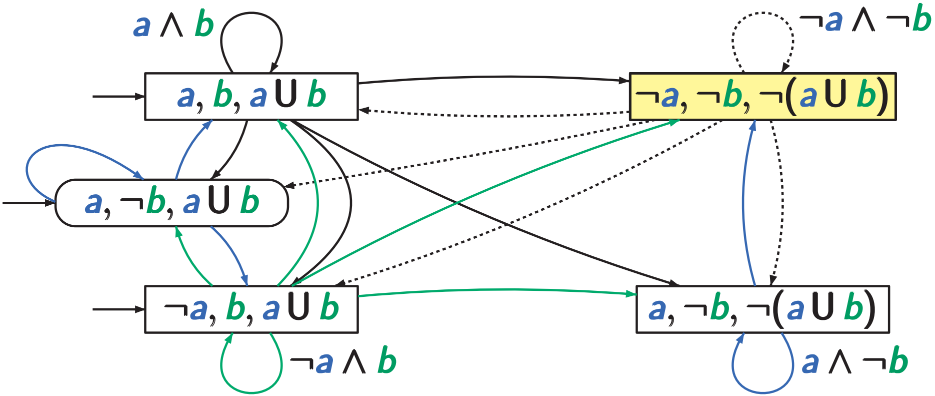
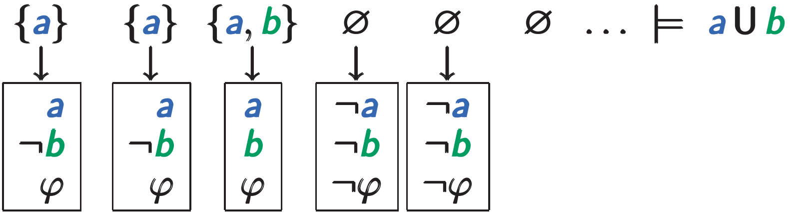


Example: (G)NBA for $\varphi = a \cup b$

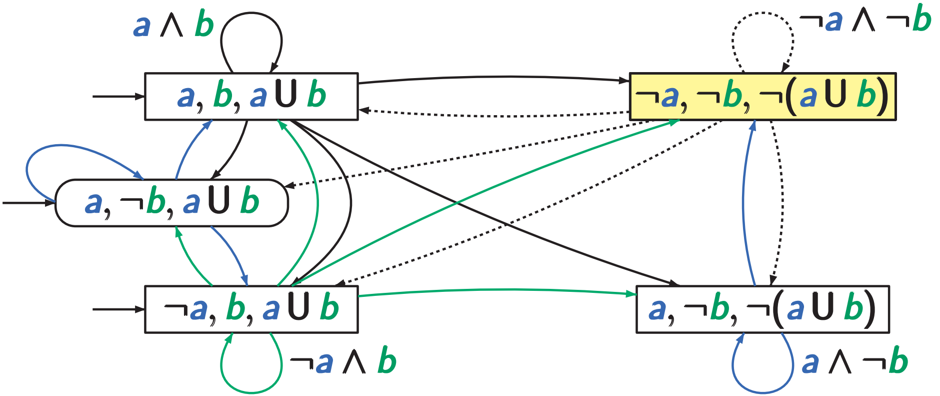


Example: (G)NBA for $\varphi = a \,\mathsf{U}\, b$





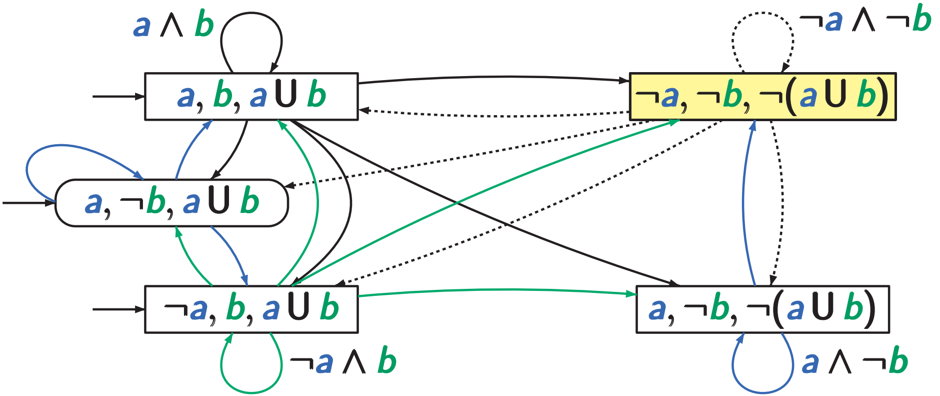
Example: (G)NBA for $\varphi = a \,\mathsf{U}\, b$

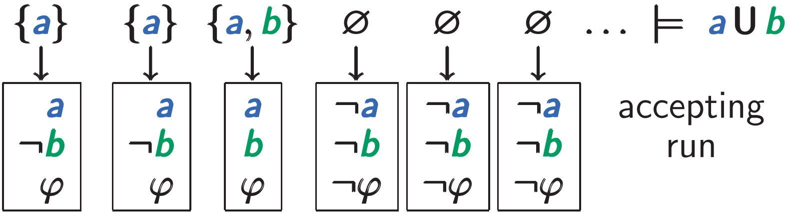


$\{a\} \quad \{a\} \quad \{a, b\} \quad \varnothing \quad \varnothing \quad \varnothing \quad \ldots \models a \,\mathsf{U}\, b$

$\{a\}$	$\{a\}$	$\{a,b\}$	$\varnothing$	$\varnothing$	$\varnothing$
a	a	a	$\neg a$	$\neg a$	$\neg a$
$\neg b$	$\neg b$	b	$\neg b$	$\neg b$	$\neg b$
φ	φ	φ	$\neg\varphi$	$\neg\varphi$	$\neg\varphi$

Example: (G)NBA for $\varphi = a \,\mathsf{U}\, b$





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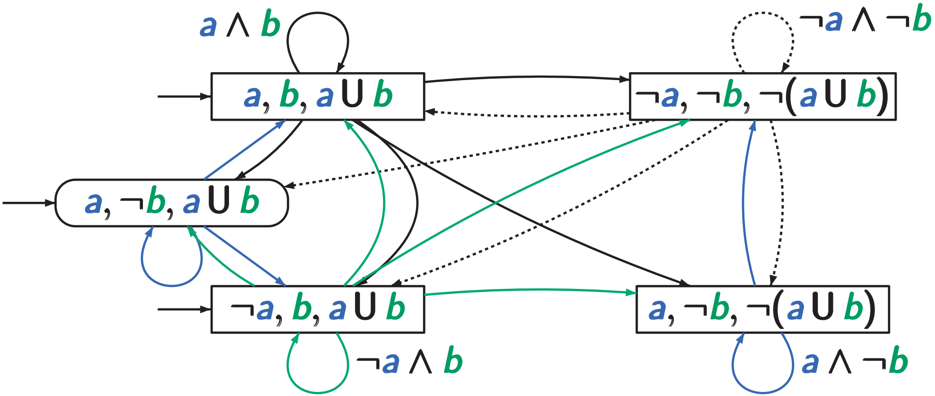

$\{a\}\,\{a\}\,\{a\}\,\{a\}\,\ldots \not\models \varphi$

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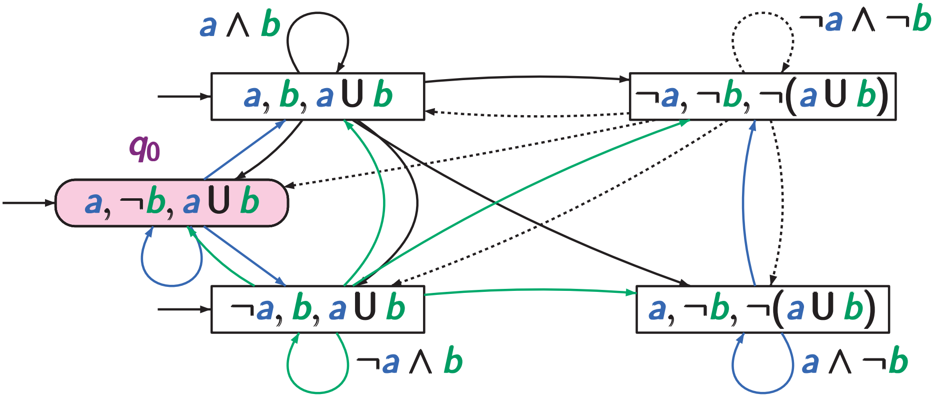

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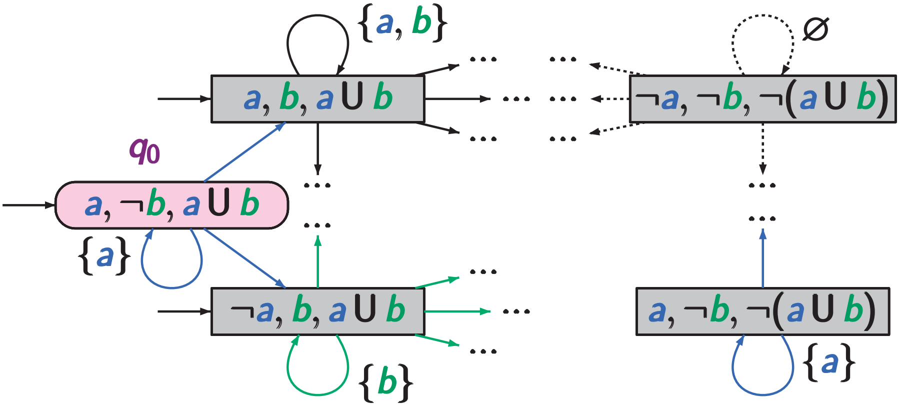
only 1 infinite run: $q_0\, q_0\, q_0\, \ldots$

Example: (G)NBA for $\varphi = a \mathsf{U} b$



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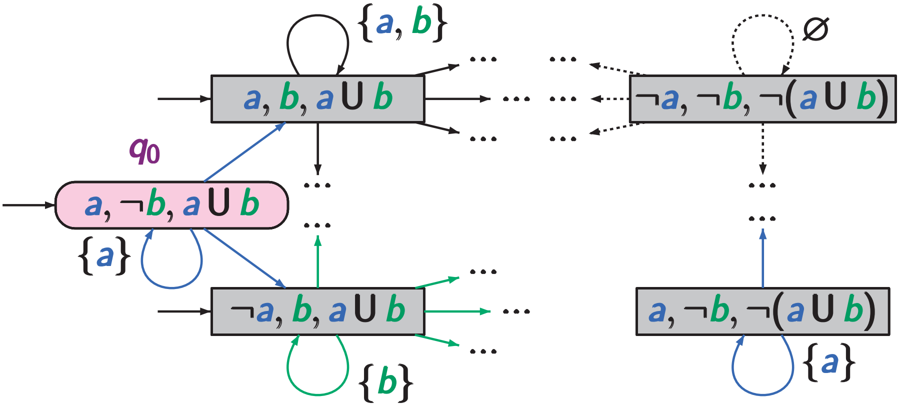

$\{a\}\,\{a\}\,\{a\}\,\{a\}\,\ldots \not\models \varphi$

only **1** infinite run: $q_0\, q_0\, q_0\, \ldots$ not accepting

GNBA for LTL-formula φ

$$\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$$

state space: $Q = \{B \subseteq cl(\varphi) : B \text{ is elementary}\}$

initial states: $Q_0 = \{B \in Q : \varphi \in B\}$

transition relation: for $B \in Q$ and $A \in 2^{AP}$:

if $A \neq B \cap AP$ then $\delta(B, A) = \varnothing$

if $A = B \cap AP$ then $\delta(B, A) =$ set of all $B' \in Q$ s.t.

$$\bigcirc\psi \in B \quad \text{iff} \quad \psi \in B'$$

$$\psi_1 \mathsf{U} \psi_2 \in B \quad \text{iff} \quad (\psi_2 \in B) \vee (\psi_1 \in B \wedge \psi_1 \mathsf{U} \psi_2 \in B')$$

acceptance set $\mathcal{F} = \{F_{\psi_1 \mathsf{U} \psi_2} : \psi_1 \mathsf{U} \psi_2 \in cl(\varphi)\}$

where $F_{\psi_1 \mathsf{U} \psi_2} = \{B \in Q : \psi_1 \mathsf{U} \psi_2 \notin B \vee \psi_2 \in B\}$