Soundness

.... of the construction LTL formula $\varphi \rightsquigarrow$ GNBA $\mathcal{G}$

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Let φ be an LTL-formula and $\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$ be the constructed GNBA.

Claim: $\mathit{Words}(\varphi) = \mathcal{L}_\omega(\mathcal{G})$

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"$\subseteq$" show: each infinite word $A_0\, A_1\, A_2 \ldots \in (2^{AP})^\omega$

with $A_0\, A_1\, A_2 \ldots \models \varphi$

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"$\supseteq$" show: for all infinite words $A_0\, A_1\, A_2 \ldots \in \mathcal{L}_\omega(\mathcal{G})$: $A_0\, A_1\, A_2 \ldots \models \varphi$

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Accepting runs for the elements of *Words*(φ)

LTL formula $\varphi \rightsquigarrow$ GNBA $\mathcal{G}$ for $\mathit{Words}(\varphi)$

states of $\mathcal{G}$ $\quad\widehat{=}\quad$ elementary formula-sets $B \subseteq \mathit{cl}(\varphi)$

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Example: $\varphi \;=\; a\, \mathsf{U}(\neg a \wedge b)$

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$$\begin{array}{cccccccc} \{a\} & \{a\} & \{a,b\} & \{b\} & \varnothing & \varnothing & \ldots \models \varphi \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ B_0 & B_1 & B_2 & B_3 & B_4 & B_5 & \end{array}$$

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$$\begin{array}{cccccccc} \{a\} & \{a\} & \{a,b\} & \{b\} & \varnothing & \varnothing & \ldots \models \varphi \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\ B_0 & B_1 & B_2 & B_3 & B_4 & B_5 & \end{array}$$

where the B_i's are states in $\mathcal{G}$, i.e., elementary subsets of $\{a, \neg a, b, \neg b, \psi, \neg\psi, \varphi, \neg\varphi\}$

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$\{a\} \quad \{a\} \quad \{a, b\} \quad \{b\} \quad \varnothing \quad \varnothing \quad \ldots \models \varphi$

$\downarrow$

$$\begin{array}{|r|} \hline a \\ \neg b \\ \neg \psi \\ \varphi \\ \hline \end{array}$$

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$$\{a\} \quad \{a\} \quad \{a, b\} \quad \{b\} \quad \varnothing \quad \varnothing \quad \ldots \models \varphi$$

$$\downarrow \qquad \downarrow$$

a	a
$\neg b$	$\neg b$
$\neg\psi$	$\neg\psi$
φ	φ

LTL formula $\varphi \rightsquigarrow$ GNBA $\mathcal{G}$ for $\mathit{Words}(\varphi)$

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$\{a\} \quad \{a\} \quad \{a, b\} \quad \{b\} \quad \varnothing \quad \varnothing \quad \ldots \quad \models \varphi$

$\{a\}$ ↓	$\{a\}$ ↓	$\{a, b\}$ ↓
a	a	a
$\neg b$	$\neg b$	b
$\neg\psi$	$\neg\psi$	$\neg\psi$
φ	φ	φ

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$\{a\}$	$\{a\}$	$\{a, b\}$	$\{b\}$	$\varnothing$	$\varnothing$	$\ldots$	$\models \varphi$
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$				
a	a	a	$\neg a$				
$\neg b$	$\neg b$	b	b				
$\neg \psi$	$\neg \psi$	$\neg \psi$	ψ				
φ	φ	φ	φ				

Accepting runs for the elements of $\mathit{Words}(\varphi)$

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Example: $\varphi \;=\; a\, \mathsf{U}(\neg a \wedge b)$ $\qquad \psi = \neg a \wedge b$

$\{a\}$	$\{a\}$	$\{a,b\}$	$\{b\}$	$\varnothing$	$\varnothing$	$\ldots$	$\models \varphi$
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$			
a	a	a	$\neg a$	$\neg a$			
$\neg b$	$\neg b$	b	b	$\neg b$			
$\neg \psi$	$\neg \psi$	$\neg \psi$	ψ	$\neg \psi$			
φ	φ	φ	φ	$\neg \varphi$			

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$\{a\}$	$\{a\}$	$\{a, b\}$	$\{b\}$	$\varnothing$	$\varnothing$	$\ldots$	$\models \varphi$
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$		
a	a	a	$\neg a$	$\neg a$	$\neg a$		
$\neg b$	$\neg b$	b	b	$\neg b$	$\neg b$		
$\neg\psi$	$\neg\psi$	$\neg\psi$	ψ	$\neg\psi$	$\neg\psi$	$\ldots$	
φ	φ	φ	φ	$\neg\varphi$	$\neg\varphi$		

GNBA for LTL-formula φ

$\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$

state space: $Q = \{B \subseteq cl(\varphi) : B \text{ is elementary}\}$

initial states: $Q_0 = \{B \in Q : \varphi \in B\}$

transition relation: for $B \in Q$ and $A \in 2^{AP}$:

if $A \neq B \cap AP$ then $\delta(B, A) = \varnothing$

if $A = B \cap AP$ then $\delta(B, A) =$ set of all $B' \in Q$ s.t.

$$\bigcirc\psi \in B \quad \text{iff} \quad \psi \in B'$$

$$\psi_1 \mathsf{U} \psi_2 \in B \quad \text{iff} \quad (\psi_2 \in B) \vee (\psi_1 \in B \wedge \psi_1 \mathsf{U} \psi_2 \in B')$$

acceptance set $\mathcal{F} = \{F_{\psi_1 \mathsf{U} \psi_2} : \psi_1 \mathsf{U} \psi_2 \in cl(\varphi)\}$

where $F_{\psi_1 \mathsf{U} \psi_2} = \{B \in Q : \psi_1 \mathsf{U} \psi_2 \notin B \vee \psi_2 \in B\}$

Elementary formula-sets

$B \subseteq cl(\varphi)$ is elementary iff:

(i) B is maximal consistent w.r.t. prop. logic,
i.e., if $\psi, \psi_1 \wedge \psi_2 \in cl(\varphi)$ then:

$$\begin{array}{rcl} \psi \notin B & \text{iff} & \neg\psi \in B \\ \psi_1 \wedge \psi_2 \in B & \text{iff} & \psi_1 \in B \text{ and } \psi_2 \in B \\ \textit{true} \in cl(\varphi) & \text{implies} & \textit{true} \in B \end{array}$$

(ii) B is locally consistent with respect to until $\mathbf{U}$,
i.e., if $\psi_1 \mathbf{U} \psi_2 \in cl(\varphi)$ then:

if $\psi_1 \mathbf{U} \psi_2 \in B$ and $\psi_2 \notin B$ then $\psi_1 \in B$

if $\psi_2 \in B$ then $\psi_1 \mathbf{U} \psi_2 \in B$

Let φ be an LTL-formula and $\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$ be the constructed GNBA.

Claim: $\mathit{Words}(\varphi) = \mathcal{L}_\omega(\mathcal{G})$

"$\subseteq$" show: each infinite word $A_0\, A_1\, A_2 \ldots \in (2^{AP})^\omega$

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Proof of $\mathcal{L}_\omega(\mathcal{G}) \subseteq Words(\varphi)$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

$$\forall F \in \mathcal{F} \ \overset{\infty}{\exists} j \geq 0.\, B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

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Proof of $\mathcal{L}_\omega(\mathcal{G}) \subseteq \mathit{Words}(\varphi)$

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as $B_0 \in Q_0$

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$$\forall F \in \mathcal{F} \ \overset{\infty}{\exists} j \geq 0.\ B_j \in F \qquad (*)$$

then for all formulas $\psi \in cl(\varphi)$:

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as $B_0 \in Q_0$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

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then for all formulas $\psi \in cl(\varphi)$:

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and $(*)$ holds

$\Longrightarrow$ $\sigma = A_0\, A_1\, A_2 \ldots \models \varphi$

Proof of $\mathcal{L}_\omega(\mathcal{G}) \subseteq Words(\varphi)$

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Proof by structural induction on ψ

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

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Proof by structural induction on ψ

base of induction:

$\psi = \mathit{true}$

$\psi = a \in AP$

Proof of $\mathcal{L}_\omega(\mathcal{G}) \subseteq Words(\varphi)$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

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then for all formulas $\psi \in cl(\varphi)$:

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Proof by structural induction on ψ

base of induction:

$\psi = \mathit{true}$

$\psi = a \in AP$

induction step:

$\psi = \neg\psi'$

$\psi = \psi_1 \wedge \psi_2$

$\psi = \bigcirc\psi'$

$\psi = \psi_1 \,\mathsf{U}\, \psi_2$

Base of induction

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

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Base of induction:

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Base of induction:

Suppose $\psi = \textit{true} \in cl(\varphi)$.

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

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Base of induction:

Suppose $\psi = \textit{true} \in cl(\varphi)$. Then $\textit{true} \in B_0$

note: ***true*** is contained in all elementary formula-sets

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

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then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \ldots \models \psi$$

Base of induction:

Suppose $\psi = \textit{true} \in cl(\varphi)$. Then $\textit{true} \in B_0$ and
$A_0 A_1 A_2 \ldots \models \textit{true}$

note: ***true*** is contained in all elementary formula-sets
true holds for all paths/traces

Claim: If $B_0 \overset{A_0}{\rightarrow} B_1 \overset{A_1}{\rightarrow} B_2 \overset{A_2}{\rightarrow} \ldots$ is a path in $\mathcal{G}$ s.t.

$$\forall F \in \mathcal{F} \ \overset{\infty}{\exists} j \geq 0.\ B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0\, A_1\, A_2 \ldots \models \psi$$

Base of induction:

Suppose $\psi = \mathit{true} \in cl(\varphi)$. Then $\mathit{true} \in B_0$ and $A_0\, A_1\, A_2 \ldots \models \mathit{true}$

Let $\psi = a \in AP$.

Base of induction

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

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$a \in B_0$

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$$a \in B_0 \iff a \in A_0$$

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Suppose $\psi = true \in cl(\varphi)$. Then $true \in B_0$ and

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$$a \in B_0 \iff a \in A_0 \iff A_0\, A_1\, A_2 \ldots \models a$$

Induction step: negation

Induction step: negation

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then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 \, A_1 \, A_2 \ldots \models \psi$$

Induction step: for $\psi = \neg\psi'$:

Induction step: negation

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

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Induction step: for $\psi = \neg\psi'$:

$$\psi \in B_0$$

iff $\psi' \notin B_0$ (maximal consistency)

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

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iff $A_0\, A_1\, A_2 \ldots \not\models \psi'$ (induction hypothesis)

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iff $A_0 A_1 A_2 \ldots \not\models \psi'$ (induction hypothesis)

iff $A_0 A_1 A_2 \ldots \models \psi$ (semantics of $\neg$)

Elementary formula-sets

$B \subseteq cl(\varphi)$ is elementary iff:

(i) B is maximal consistent w.r.t. prop. logic, i.e., if ψ, $\psi_1 \wedge \psi_2 \in cl(\varphi)$ then:

$$\psi \notin B \quad \text{iff} \quad \neg\psi \in B$$

$$\psi_1 \wedge \psi_2 \in B \quad \text{iff} \quad \psi_1 \in B \text{ and } \psi_2 \in B$$

$$\mathit{true} \in cl(\varphi) \quad \text{implies} \quad \mathit{true} \in B$$

(ii) B is locally consistent with respect to until $\mathbf{U}$, i.e., if $\psi_1 \,\mathbf{U}\, \psi_2 \in cl(\varphi)$ then:

if $\psi_1 \,\mathbf{U}\, \psi_2 \in B$ and $\psi_2 \notin B$ then $\psi_1 \in B$

if $\psi_2 \in B$ then $\psi_1 \,\mathbf{U}\, \psi_2 \in B$

$B \subseteq cl(\varphi)$ is elementary iff:

(i) B is maximal consistent w.r.t. prop. logic, i.e., if ψ, $\psi_1 \wedge \psi_2 \in cl(\varphi)$ then:

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$$\textit{true} \in cl(\varphi) \quad \text{implies} \quad \textit{true} \in B$$

(ii) B is locally consistent with respect to until **U**, i.e., if $\psi_1 \mathbf{U} \psi_2 \in cl(\varphi)$ then:

if $\psi_1 \mathbf{U} \psi_2 \in B$ and $\psi_2 \notin B$ then $\psi_1 \in B$

if $\psi_2 \in B$ then $\psi_1 \mathbf{U} \psi_2 \in B$

Induction step: conjunction

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

$$\forall F \in \mathcal{F} \ \overset{\infty}{\exists} j \geq 0.\ B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0\,A_1\,A_2 \ldots \models \psi$$

Induction step: for $\psi = \psi_1 \wedge \psi_2$

Induction step: conjunction

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

$$\forall F \in \mathcal{F} \ \overset{\infty}{\exists} j \geq 0.\ B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0\, A_1\, A_2 \ldots \models \psi$$

Induction step: for $\psi = \psi_1 \wedge \psi_2$

$$\psi \in B_0$$

Induction step: conjunction

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

$$\forall F \in \mathcal{F} \ \overset{\infty}{\exists} j \geq 0.\, B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0 A_1 A_2 \ldots \models \psi$$

Induction step: for $\psi = \psi_1 \wedge \psi_2$

$\psi \in B_0$

iff $\psi_1, \psi_2 \in B_0$ (maximal consistency)

Induction step: conjunction

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

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Induction step: for $\psi = \psi_1 \wedge \psi_2$

$\psi \in B_0$

iff $\psi_1, \psi_2 \in B_0$ (maximal consistency)

iff $A_0\, A_1\, A_2 \ldots \models \psi_1$ and $A_0\, A_1\, A_2 \ldots \models \psi_2$ (IH)

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

$$\forall F \in \mathcal{F} \ \overset{\infty}{\exists} j \geq 0.\ B_j \in F$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0\, A_1\, A_2 \ldots \models \psi$$

Induction step: for $\psi = \psi_1 \wedge \psi_2$

$\psi \in B_0$

iff $\psi_1, \psi_2 \in B_0$ (maximal consistency)

iff $A_0\, A_1\, A_2 \ldots \models \psi_1$ and $A_0\, A_1\, A_2 \ldots \models \psi_2$ (IH)

iff $A_0\, A_1\, A_2 \ldots \models \psi$ (semantics of $\wedge$)

Induction step: next step

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

$$\forall F \in \mathcal{F} \ \overset{\infty}{\exists} j \geq 0.\ B_j \in F$$

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Induction step: for $\psi = \bigcirc \psi'$:

GNBA for LTL-formula φ

$$\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$$

state space: $Q = \{ B \subseteq cl(\varphi) : B \text{ is elementary} \}$

initial states: $Q_0 = \{ B \in Q : \varphi \in B \}$

transition relation: for $B \in Q$ and $A \in 2^{AP}$:

if $A \neq B \cap AP$ then $\delta(B, A) = \varnothing$

if $A = B \cap AP$ then $\delta(B, A) =$ set of all $B' \in Q$ s.t.

$$\bigcirc \psi \in B \quad \text{iff} \quad \psi \in B'$$

$$\psi_1 \, \mathsf{U} \, \psi_2 \in B \quad \text{iff} \quad (\psi_2 \in B) \vee (\psi_1 \in B \wedge \psi_1 \, \mathsf{U} \, \psi_2 \in B')$$

acceptance set $\mathcal{F} = \{ F_{\psi_1 \mathsf{U} \psi_2} : \psi_1 \, \mathsf{U} \, \psi_2 \in cl(\varphi) \}$

where $F_{\psi_1 \mathsf{U} \psi_2} = \{ B \in Q : \psi_1 \, \mathsf{U} \, \psi_2 \notin B \vee \psi_2 \in B \}$

Induction step: next step

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

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Induction step: for $\psi = \bigcirc \psi'$:

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Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

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Induction step: for $\psi = \bigcirc \psi'$:

$$\psi \in B_0$$

Induction step: next step

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

$$\forall F \in \mathcal{F} \ \overset{\infty}{\exists} j \geq 0.\ B_j \in F$$

$$B_1 \in \delta(B_0, A_0)$$

then for all formulas $\psi \in cl(\varphi)$:

$$\psi \in B_0 \quad \text{iff} \quad A_0\, A_1\, A_2 \ldots \models \psi$$

Induction step: for $\psi = \bigcirc \psi'$:

$$\psi \in B_0$$

iff $\psi' \in B_1$ (definition of δ)

Induction step: next step

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

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Induction step: for $\psi = \bigcirc \psi'$:

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iff $\psi' \in B_1$ (definition of δ)

iff $A_1\, A_2\, A_3 \ldots \models \psi'$ (induction hypothesis)

Induction step: next step

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

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iff $A_1\, A_2\, A_3 \ldots \models \psi'$ (induction hypothesis)

iff $A_0\, A_1\, A_2\, A_3 \ldots \models \psi$ (semantics of $\bigcirc$)

Induction step: until

$B \subseteq cl(\varphi)$ is elementary iff:

(i) B is maximal consistent w.r.t. prop. logic,
i.e., if ψ, $\psi_1 \wedge \psi_2 \in cl(\varphi)$ then:

$\psi \notin B$	iff	$\neg\psi \in B$
$\psi_1 \wedge \psi_2 \in B$	iff	$\psi_1 \in B$ and $\psi_2 \in B$
$true \in cl(\varphi)$	implies	$true \in B$

(ii) B is locally consistent with respect to until **U**,
i.e., if $\psi_1 \mathbf{U} \psi_2 \in cl(\varphi)$ then:

if $\psi_1 \mathbf{U} \psi_2 \in B$ and $\psi_2 \notin B$ then $\psi_1 \in B$
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Recall: GNBA for LTL-formula φ

$$\mathcal{G} = (Q, 2^{AP}, \delta, Q_0, \mathcal{F})$$

state space: $Q = \{B \subseteq cl(\varphi) : B \text{ is elementary}\}$

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if $A = B \cap AP$ then $\delta(B, A) =$ set of all $B' \in Q$ s.t.

$$\bigcirc\psi \in B \quad \text{iff} \quad \psi \in B'$$

$$\psi_1 \mathsf{U} \psi_2 \in B \quad \text{iff} \quad (\psi_2 \in B) \vee (\psi_1 \in B \wedge \psi_1 \mathsf{U} \psi_2 \in B')$$

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Recall: GNBA for LTL-formula φ

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Induction step: until

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

$\forall F \in \mathcal{F} \ \overset{\infty}{\exists} j \geq 0.\, B_j \in F$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0\, A_1\, A_2 \ldots \models \psi$

Induction step for $\psi = \psi_1 \,\mathsf{U}\, \psi_2$:

Induction step: until (part "$\Longleftarrow$")

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

$$\forall F \in \mathcal{F} \ \overset{\infty}{\exists} j \geq 0.\ B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0\, A_1\, A_2 \ldots \models \psi$

Induction step for $\psi = \psi_1 \,\mathsf{U}\, \psi_2$:

"$\Longleftarrow$": Suppose $A_0\, A_1\, A_2 \ldots \models \psi$.

Induction step: until (part "$\Longleftarrow$")

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

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Induction step for $\psi = \psi_1 \mathbf{U}\, \psi_2$:

"$\Longleftarrow$": Suppose $A_0\, A_1\, A_2 \ldots \models \psi$. Let $j \geq 0$ s.t.

$$
\begin{array}{ll}
A_j\, A_{j+1}\, A_{j+2} \ldots & \models \psi_2 \\
A_{j-1}\, A_j\, A_{j-1} \ldots & \models \psi_1 \\
A_{j-2}\, A_{j-1}\, A_j \ldots & \models \psi_1 \\
\vdots & \\
A_0\, A_1\, A_2\, A_3 \ldots & \models \psi_1
\end{array}
$$

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

$$\forall F \in \mathcal{F} \ \overset{\infty}{\exists} j \geq 0.\ B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0\, A_1\, A_2 \ldots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U}\, \psi_2$:

"$\Longleftarrow$": Suppose $A_0\, A_1\, A_2 \ldots \models \psi$. Let $j \geq 0$ s.t.

$$\begin{array}{lcl}
A_j\, A_{j+1}\, A_{j+2} \ldots \models \psi_2 & \overset{\text{IH}}{\Rightarrow} & \psi_2 \in B_j \\
A_{j-1}\, A_j\, A_{j-1} \ldots \models \psi_1 & \Rightarrow & \psi_1 \in B_{j-1} \\
A_{j-2}\, A_{j-1}\, A_j \ldots \models \psi_1 & \Rightarrow & \psi_1 \in B_{j-2} \\
\vdots & & \vdots \\
A_0\, A_1\, A_2\, A_3 \ldots \models \psi_1 & \Rightarrow & \psi_1 \in B_0
\end{array}$$

Induction step: until (part "$\Longleftarrow$")

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

$$\forall F \in \mathcal{F} \ \overset{\infty}{\exists} j \geq 0.\ B_j \in F$$

B_j is elementary

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0\, A_1\, A_2 \ldots \models \psi$

Induction step for $\psi = \psi_1 \,\mathsf{U}\, \psi_2$:

"$\Longleftarrow$": Suppose $A_0\, A_1\, A_2 \ldots \models \psi$. Let $j \geq 0$ s.t.

$$\begin{array}{lllll}
A_j\, A_{j+1}\, A_{j+2} \ldots & \models \psi_2 & \overset{\text{IH}}{\Rightarrow} & \psi_2 \in B_j & \Rightarrow \ \psi \in B_j \\
A_{j-1}\, A_j\, A_{j-1} \ldots & \models \psi_1 & \Rightarrow & \psi_1 \in B_{j-1} & \\
A_{j-2}\, A_{j-1}\, A_j \ldots & \models \psi_1 & \Rightarrow & \psi_1 \in B_{j-2} & \\
\vdots & & & \vdots & \\
A_0\, A_1\, A_2\, A_3 \ldots & \models \psi_1 & \Rightarrow & \psi_1 \in B_0 &
\end{array}$$

Induction step: until (part "$\Longleftarrow$")

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

$$\forall F \in \mathcal{F} \ \overset{\infty}{\exists} j \geq 0.\ B_j \in F \qquad B_j \in \delta(B_{j-1}, A_{j-1})$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0\,A_1\,A_2 \ldots \models \psi$

Induction step for $\psi = \psi_1 \mathsf{U} \psi_2$:

"$\Longleftarrow$": Suppose $A_0\,A_1\,A_2 \ldots \models \psi$. Let $j \geq 0$ s.t.

$$
\begin{array}{lllll}
A_j\,A_{j+1}\,A_{j+2} \ldots \models \psi_2 & \overset{\text{IH}}{\Rightarrow} & \psi_2 \in B_j & \Rightarrow & \psi \in B_j \\
A_{j-1}\,A_j\,A_{j-1} \ldots \models \psi_1 & \Rightarrow & \psi_1 \in B_{j-1} & \wedge & \psi \in B_{j-1} \\
A_{j-2}\,A_{j-1}\,A_j \ldots \models \psi_1 & \Rightarrow & \psi_1 \in B_{j-2} & & \\
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Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

$$\forall F \in \mathcal{F} \ \overset{\infty}{\exists} j \geq 0.\ B_j \in F \qquad B_{j-1} \in \delta(B_{j-2}, A_{j-2})$$

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Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

"$\Longleftarrow$": Suppose $A_0\, A_1\, A_2 \ldots \models \psi$. Let $j \geq 0$ s.t.

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A_{j-2}\, A_{j-1}\, A_j \ldots & \models \psi_1 & \Rightarrow & \psi_1 \in B_{j-2} & \wedge & \psi \in B_{j-2} \\
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A_{j-2}\, A_{j-1}\, A_j \ldots & \models \psi_1 & \Rightarrow & \psi_1 \in B_{j-2} & \wedge & \psi \in B_{j-2} \\
\vdots & & & \vdots & \vdots & \\
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$$\left.\begin{array}{l} \quad \psi \in B_0 \wedge \psi_2 \notin B_0 \\ \Rightarrow \psi \in B_1 \wedge \psi_2 \notin B_1 \\ \Rightarrow \psi \in B_2 \wedge \psi_2 \notin B_2 \\ \qquad\qquad \vdots \end{array}\right\} \Longrightarrow \forall j \geq 0.\ B_j \notin F_\psi \text{ where}$$

$$F_\psi = \{B : \psi \notin B \text{ or } \psi_2 \in B\}$$

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Contradiction!

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$\neg\psi_2,\ \psi \in B_0$ $\longleftarrow$ by assumption

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\vdots & \vdots & \vdots & \vdots \\
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\vdots & \vdots & \vdots & \vdots \\
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$$\begin{array}{rcl} & \overset{\text{IH}}{\Longrightarrow} & A_j\, A_{j+1} \ldots \models \psi_2 \\ \neg\psi_2, \psi_1, \psi \in B_{j-1} & \Longrightarrow & A_{j-1}\, A_j \ldots \models \psi_1 \\ \vdots & \vdots & \vdots \qquad \vdots \\ \neg\psi_2, \psi_1, \psi \in B_0 & \Longrightarrow & A_0\, A_1\, A_2 \ldots \models \psi_1 \\ & & \Downarrow \end{array}$$

Induction step: until (part "$\Longrightarrow$")

Claim: If $B_0 \xrightarrow{A_0} B_1 \xrightarrow{A_1} B_2 \xrightarrow{A_2} \ldots$ is a path in $\mathcal{G}$ s.t.

$$\forall F \in \mathcal{F} \ \overset{\infty}{\exists} j \geq 0.\ B_j \in F$$

then for all $\psi \in cl(\varphi)$: $\psi \in B_0$ iff $A_0\, A_1\, A_2 \ldots \models \psi$

Induction step for $\psi = \psi_1 \mathbf{U} \psi_2$:

Let $\psi \in B_0$ and $j \geq 0$ minimal s.t. $\psi_2 \in B_j$

$$\begin{array}{rcl} & \overset{\text{IH}}{\Longrightarrow} & A_j\, A_{j+1} \ldots \models \psi_2 \\ \neg\psi_2, \psi_1, \psi \in B_{j-1} & \Longrightarrow & A_{j-1}\, A_j \ldots \models \psi_1 \\ \vdots & \vdots & \vdots \\ \neg\psi_2, \psi_1, \psi \in B_0 & \Longrightarrow & A_0\, A_1\, A_2 \ldots \models \psi_1 \\ & & \Downarrow \\ & & A_0\, A_1\, A_2 \ldots \models \psi = \psi_1 \mathbf{U} \psi_2 \end{array}$$

Complexity: LTL $\rightsquigarrow$ NBA

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For each **LTL** formula φ, there is an **NBA** $\mathcal{A}$ s.t.

$$\mathcal{L}_\omega(\mathcal{A}) \quad = \quad \mathit{Words}(\varphi)$$

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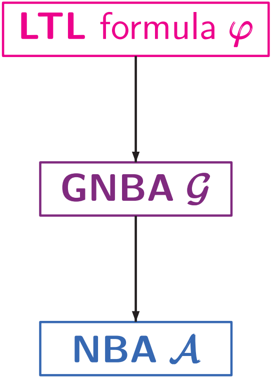

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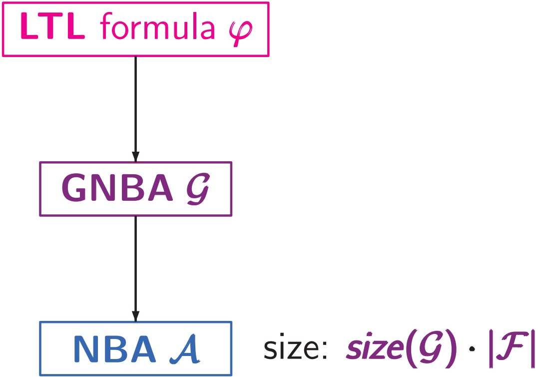

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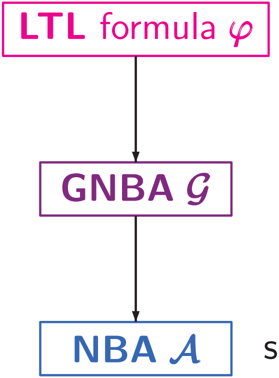

size: $\mathit{size}(\mathcal{G}) \cdot |\mathcal{F}|$

$|\mathcal{F}|$ = number of acceptance sets in $\mathcal{G}$

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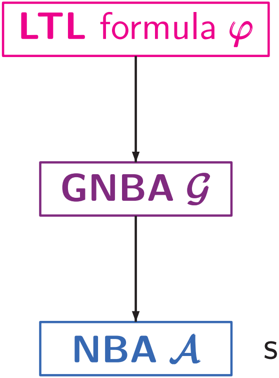

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$$\mathcal{L}_\omega(\mathcal{A}) \;=\; \mathit{Words}(\varphi)$$

LTL formula φ

↓

GNBA $\mathcal{G}$ — size: $2^{|cl(\varphi)|}$

↓

NBA $\mathcal{A}$ — size: $\mathit{size}(\mathcal{G}) \cdot |\mathcal{F}|$

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For each **LTL** formula φ, there is an **NBA** $\mathcal{A}$ s.t.

$$\mathcal{L}_\omega(\mathcal{A}) = \mathit{Words}(\varphi) \text{ and}$$
$$\mathit{size}(\mathcal{A}) \leq 2^{|cl(\varphi)|} \cdot |\varphi|$$

LTL formula φ

↓

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$$\mathcal{L}_\omega(\mathcal{A}) = \mathit{Words}(\varphi) \text{ and}$$

$$\mathit{size}(\mathcal{A}) \leq 2^{|cl(\varphi)|} \cdot |\varphi| = 2^{\mathcal{O}(|\varphi|)}$$

LTL formula φ

↓

GNBA $\mathcal{G}$ — size: $2^{|cl(\varphi)|}$

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Size of NBA for LTL formulas

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For the proposed transformation **LTL** $\rightsquigarrow$ **NBA**:

> The constructed NBA for LTL formulas are often unnecessarily complicated

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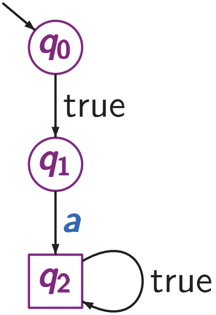
NBA for $\bigcirc a$



constructed GNBA has **4** states and **8** edges

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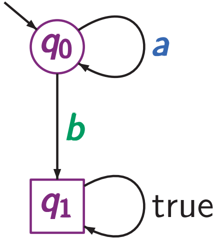
NBA for $a \, \mathbf{U} \, b$



constructed (G)NBA has **5** states and **20** edges

Size of NBA for LTL formulas

For the proposed transformation **LTL** $\rightsquigarrow$ **NBA**:

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... but there exists LTL formulas φ_n such that

- $|\varphi_n| = \mathcal{O}(poly(n))$
- each NBA for φ_n has at least 2^n states