

Notation

$$ax \leq b \quad ax = b$$

$$\begin{matrix} [a_1 \dots a_n] \\ 1 \times n \end{matrix} \begin{matrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \\ n \times 1 \end{matrix} \leq b$$

$$a_1, \dots, a_n, b \in \mathbb{Q}$$

$$x_1, \dots, x_n \in \mathbb{R}$$

$$\sum_{i=1}^n a_i x_i \leq b$$

$$Ax \leq b \quad Ax = b$$

$$\begin{matrix} \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots \\ a_{m1} & \dots & a_{mn} \end{bmatrix} \\ m \times n \end{matrix} \begin{matrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \\ n \times 1 \end{matrix} \leq \begin{matrix} \begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix} \\ m \times 1 \end{matrix}$$

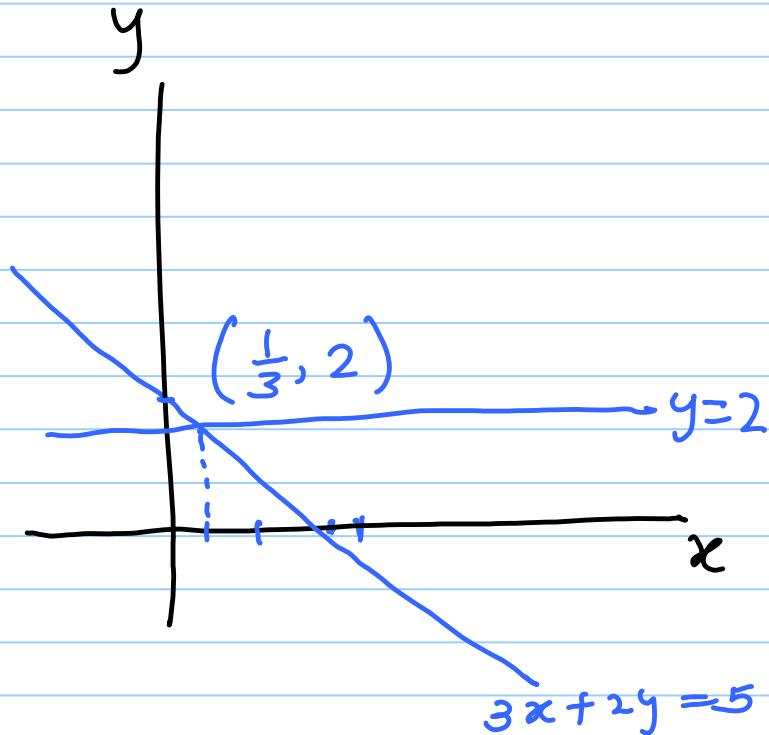
$$\bigwedge_{i=1}^m \left(\sum_{j=1}^n a_{ij} x_j \leq b_i \right)$$

Simultaneous Linear Equations

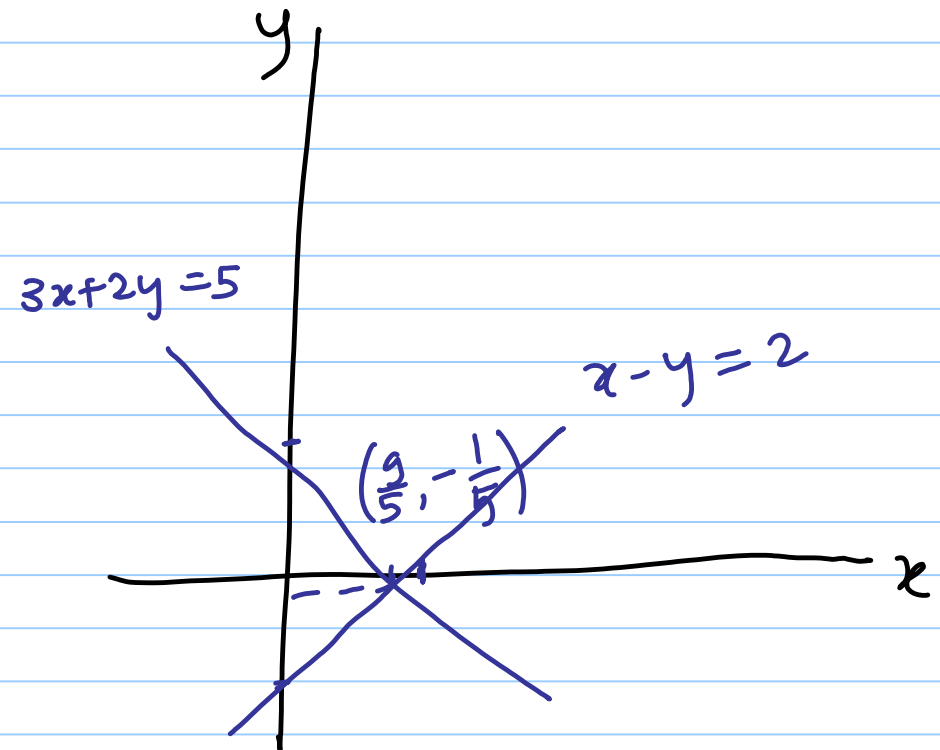
Note Title

2/19/2011

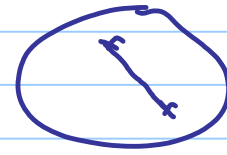
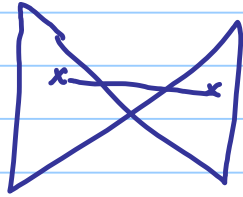
Example I. $3x + 2y = 5$
 $y = 2$



Example II. $3x + 2y = 5$
 $x - y = 2$



Convex Sets



Convex Polyhedra

Convex polyhedron: Intersection of half-spaces

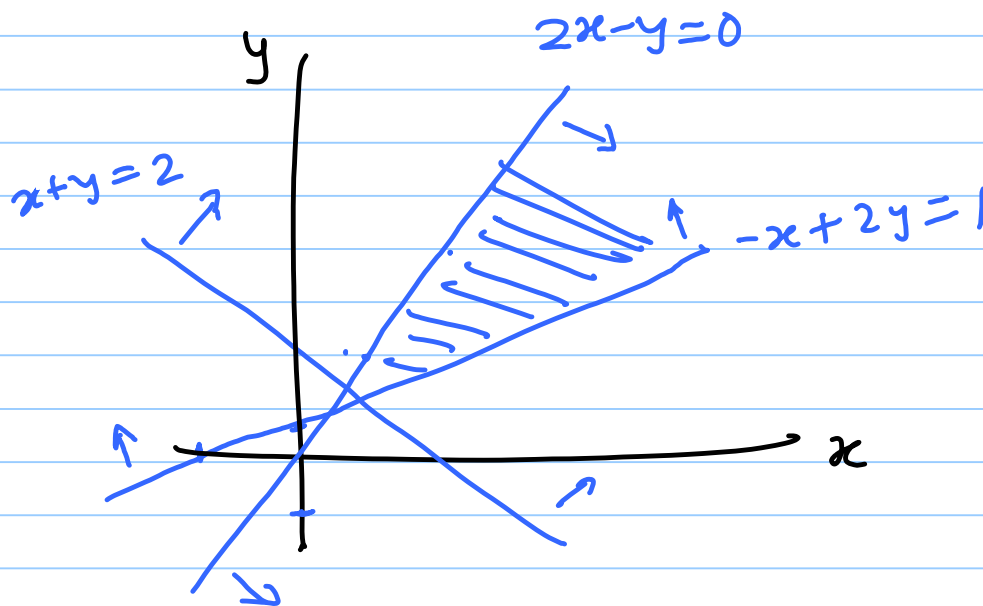
Half-space: Inequality of the form $a_1x_1 + \dots + a_nx_n \leq b_n$

Example III.

$$x + y \geq 2$$

$$2x - y \geq 0$$

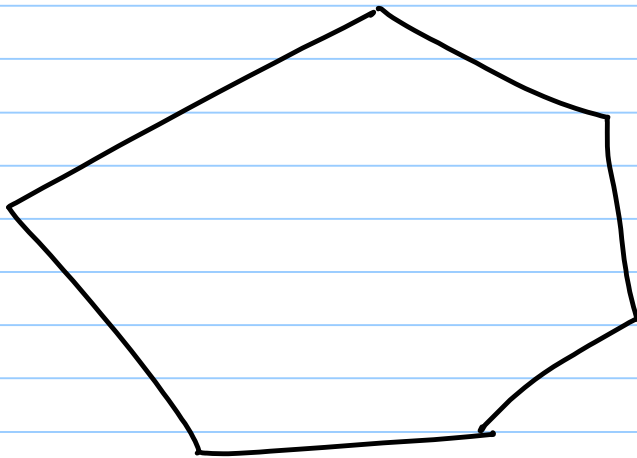
$$-x + 2y \geq 1$$



Linear Programming

$$\text{maximize } \vec{c}\vec{x} \text{ s.t. } A\vec{x} \leq \vec{b}, \vec{x} \geq \vec{0}$$

Optimal solution if it exists, is always found at a corner/edge.



Simplex (for optimization)

- 1 Find a corner of the polyhedron
- 2 Walk along an adjacent edge to a max corner, if possible

Feasibility of System of Inequalities

≡ Find a corner of the convex polyhedron

Step I.

$$x=0$$

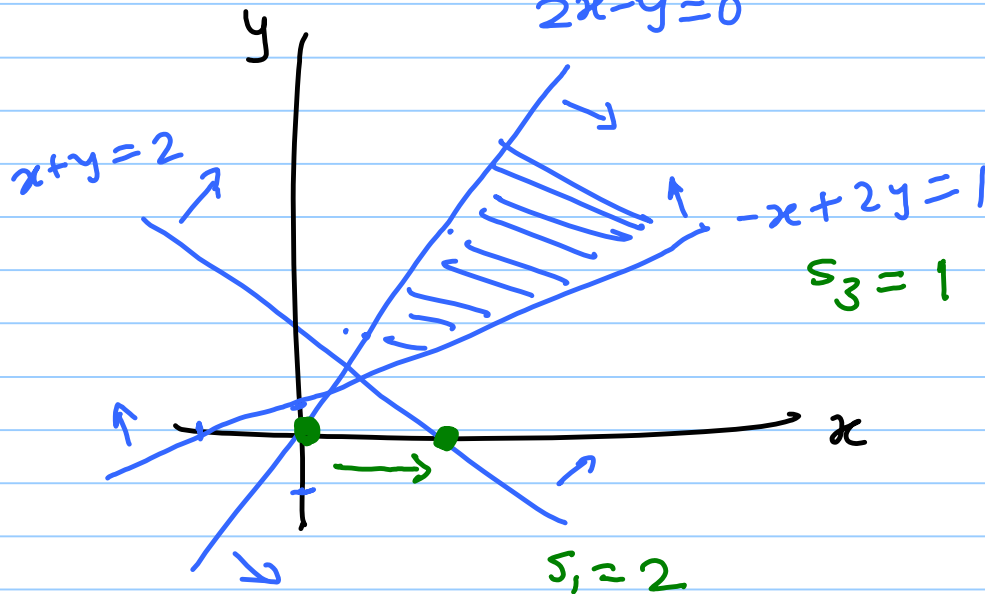
$$y=0$$

$$s_2 = 0$$
$$2x - y = 0$$

$$x + y \geq 2 \quad s_1$$

$$2x - y \geq 0 \quad s_2$$

$$-x + 2y \geq 1 \quad s_3$$



Step II. $x=2$ $y=0$

Step IV. $x=1$ $y=1$

