Soundness of Range Allocation

Observation: We ignore the boolean structure of the formula φ while constructing the ranges.

Claim: If the formula is satisfiable then there exists an assignment α with range in R s.t. $\alpha \models \varphi$.

Approach: Give an algorithm s.t. given any consistent set B of literals of φ, it returns an assignment α s.t.

[I] α is feasible i.e. $\alpha(x) \in R(x)$

[II] α satisfies the literals in B.

Feasibility of the assignment

- Each vertex is assigned a value in step I.B, II.A, or II.B
- Step 1 of the assignment algorithm covers I.B & II.A
- Step 2 of the assignment algorithm covers II.B

Hence, $\alpha(x) \in R(x)$

Satisfiability by the assignment

1. $x_i = x_j \in B$

We need to show that $\alpha(x_i) = \alpha(x_j)$.

Either both x_i and x_j are assigned their values in step 1 or both in step 2.

* Step 1: They are assigned the minimum value corresponding to the same mixed vertex x_k.

* Step 2: They are assigned the value in R assigned in II.B. We have: $R(x_i) \cap R(x_j) \neq \phi$.

2. $x_i \neq x_j \in B$

We need to show that $\alpha(x_i) \neq \alpha(x_j)$.

* If x_i and x_j both are assigned in step 1 then there must be x_k and x_k' s.t. $x_k \neq x_k'$ and $\alpha(x_i) = \alpha(x_k)$, $\alpha(x_j) = \alpha(x_k')$. Else there is a contradictory cycle in B.

* One of them is assigned a value in step 1 and another in step 2 - which are distinct by construction.

* Both assigned their values in step 2. If they are assigned the same value then they are connected by an equality path in B. Since $x_i \neq x_j \in B$ - B contains a contradictory cycle. But B is consistent.